Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 234 NOVEMBER, 1942 No. 5

CONCLUDING PAPER.*

BY

CARL VAN DOREN, Ph.D., Litt.D., L.H.D.,

Author and Editor, New York.

Mr. Staples; Ladies and Gentlemen: I am sorry I was unable to be present at the other lectures in this series. But as your Chairman told you, I have not been, even in Honolulu, too far from Franklin. For Hawaii was discovered by James Cook, a friend of Franklin. Before Europe heard of Cook's death on his last voyage, Franklin in Paris issued the famous passport which he sent to all the commanders of American naval vessels, instructing them that if they should happen to overtake Captain Cook on his way they were not to molest him or do damage to his cargo. His voyage was purely for science and it was important that his ship be allowed to return safe.

Among the speakers in this series it was not possible to include George Simpson Eddy. I would like to say a word or two about the regret we must feel that that master of all Franklinists could not be with us. Mr. Eddy for something like thirty years has been devoting many hours a day to Franklin, and has accumulated an immense amount of information about him. Mr. Eddy's writings have been chiefly confined to special topics, but his knowledge is unequalled. For example, he has prepared a whole manuscript volume

* Delivered in the Hall of The Franklin Institute, April 19, 1940. This was the closing address in the "Meet Dr. Franklin" Conference.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

merely listing the books Franklin is known to have owned. He has found out where Franklin was on every day of his life about which there is any conceivable record or reasonable conjecture. It has always been to me a triumph when now and then—but rarely—I could spring a new Franklin fact on Mr. Eddy. Being modest about his work, he is not so well-known to the general public as he should be. I should like to pay him the tribute of naming him the most erudite student of Franklin that ever lived. He knows more about Franklin than Franklin knew about himself.

I should like too to speak of the need of a new edition of Franklin's writings. There was an edition by his grandson William Temple Franklin in 1818. In 1840 Jared Sparks published another, and in 1887–1889 John Bigelow a third. In 1905–1907 Albert Henry Smyth of Philadelphia produced the best. But the Smyth edition is long out of print, and there ought to be an edition of Franklin comparable to the great edition of Washington recently completed by the late J. C. Fitzpatrick of the Library of Congress.

We do not always realize how many available Franklin pieces have never been printed in any collected edition. There are, to begin with, his verses. Some of them can be found in scattered places but all of them ought to be brought together. Franklin's first book, "Dissertation on Liberty and Necessity, Pleasure and Pain" (1725), has lately been reproduced in facsimile, but never included in a collected Franklin. The pamphlets he wrote for the Rev. Mr. Hemphill in Philadelphia in 1733 might be included. There are his surreptitious writings which it is now time, I think, to include in Franklin's collected work. His "Reflections on Courtship and Marriage" (1746) is an amusing pamphlet, and an interesting early argument for the better education of women. It has never been collected or apparently reprinted since 1758 or 1759. His "Account of the Pennsylvania Hospital" (1754) is probably the best history of any such enterprise in the English language, but it has never been collected. Verner W. Crane in his lecture in this series told of a hitherto unidentified body of political writing by Franklin, much of it published quietly over pseudonyms. These pieces, originally contributed to news-

papers and magazines, ought to be collected in a complete Franklin.

There are, too, the marginal notes that Franklin wrote in various pamphlets he examined. Some remarkable constitutional discussions of the British Empire and the relations of America to England are to be found there in Franklin's handwriting. Some of them have been sketchily reprinted, but they should all be systematically brought together. His bagatelles have not all been collected and have never been adequately edited. Many Franklin letters remain uncollected, some for no reason at all—for example the notable letter written on 17 January 1756 during his campaign against the Indians. This is to be found in the *Minutes of the Provincial Council*, but in no edition of Franklin's collected writings. There are his letters written to the Shipley family, which Smyth could not find though they had already been published in Virginia in 1888 by J. L. Payton in an obscure appendix. And there are scores—if not hundreds—of Franklin letters still hoarded by private owners.

Since I published my book on Franklin many things have come to life in such collections. The most interesting is the superb will of 1757 written by Franklin while he waited in New York for the delayed boat that was to carry him on his first diplomatic mission to England. It is of course all in his own hand, contains the lofty passage of resignation repeated from his 1750 will, and gives some valuable information about his property and his attitude toward his children and friends.

A satisfactory edition of Franklin's work would have to include a good many letters *to* him. The American Philosophical Society, the richest of Franklin collections, has an amazing number of letters from learned, enlightened, and ambitious people who wrote to him in numerous languages. He was a kind of philosophical clearing house of his age.

If there is need for a fuller edition of Franklin's writings so is there for a fuller biography. I hereby announce, for the first time in public, that I hope in 1945 to issue a revised edition of my book, half again as long as the first. I shall particularly aim to enlarge the accounts of Franklin's countless friends, and to exhibit the other side of his correspondence to show what people had written him that led him to write back

as he did. I had been thinking about the book for twenty years, and I realized that twenty years more would not be enough unless I could make a bold beginning, bring fresh materials to light, and draw other scholars into renewed work in what I call the Franklin science. No one person could do it all. It calls for coöperative activity. Franklin was so much a syndicate of men that it needs a syndicate of men to comprehend him.

Professor Carl R. Woodward, in his admirable lecture in this series, clearly proved that Franklin had no experimental farm. With that ends the hunt for a farm which never existed except in a letter thought to be Franklin's but actually Charles Read's. Professor Crane's discovery of unknown political writings by Franklin will on the other hand start a new hunt for a clearer understanding of Franklin's political ideas and activities. Julian P. Boyd's magnificent collection of "Indian Treaties Printed by Benjamin Franklin"; J. Bennett Nolan's "Benjamin Franklin in Scotland and Ireland" and "Printer Strahan's Book Account"; Carl Bridenbaugh's "Cities in the Wilderness"—a study of American cities of the 18th century; and valuable monographs by Dixon Wecter and others appearing in various learned journals cover all kinds of points and have increased or defined our knowledge of Franklin.

Perhaps some of you may be interested in the details of certain inquiries about Franklin that are still going on. The University of Pennsylvania owns a manuscript called "Elegy on My Sister Franklin." It was known to exist as far back as 1917, the earliest record I have seen, but where it was before that nobody seems to know. It looks to me like Franklin's handwriting, but the probable date is earlier than any undoubted handwriting of his that can be compared with it. It is signed "B. F." One line in it appears to parody the burlesque New England elegy which Franklin at sixteen wrote for one of his Dogood papers. The "Elegy on My Sister Franklin" must refer to a sister-in-law, the wife of one of his brothers. But who was the sister-in-law whose death led him to write an elegy? And when did she die? The Franklins were obscure people and left few records in Boston.

He had a brother, Peter Franklin, who on 2 September

1714 married Mary Harman, but she lived till after his death in 1766. Samuel Franklin, another brother, on 16 May 1705 married Elizabeth Tyng, who is not known to have died early. There was a Samuel Franklin who on 13 August 1719 married Hannah Kellineck. It has still to be found out whether this was a second marriage of Dr. Franklin's brother, or the first marriage of another Samuel, Dr. Franklin's cousin. John Franklin, a third brother, married first a Mary Gooch and then later Elizabeth Gooch Hubbart (or Hubbard) who was presumably a kinswoman of his first wife. Franklin visited this brother in Newport in 1724 and says he "had been married and settled there some years," but there is no reference in the "Autobiography" to a first and second marriage. Then did Franklin have any sister-in-law who died about 1718–1719, when he was in his poetical mood? Till we can find an occasion for it, it will be difficult to date the "Elegy" or to call it Franklin's. Mr. Eddy suggests that it may be by the poetical uncle, also Benjamin Franklin. It does not seem to me like the known poems of the uncle, but more like the work of a youngster such as Franklin was in his ballad days. This we know was about the end of 1718 or the beginning of 1719.

Or take another kind of research in which the detective technique is necessary. The Public Archives of Canada in 1939 wrote to me about an unidentified diary in their possession. It was a diary kept, they thought, by an Englishman in Paris in 1782, a traveler who was a good deal at Franklin's house. The archivist asked if I could tell who the diarist was. I had not read more than a few pages before I was almost certain it was one of the lost manuscripts of Sir Edward Newenham, the Irish patriot, a friend and correspondent of Franklin. Here were the reasons:

I knew that Newenham in October 1782 planned to go to Paris. The American Philosophical Society has a letter to William Franklin, who had just returned from New York to London, saying that Newenham was going to Paris and would take any non-political messages the son might want to send to his father in Paris. The diarist speaks of calling in Paris on his banker on 13 October to ask for letters of introduction to people in towns he would go through on his way to Geneva. On that same day, 13 October, Newenham is known to have

written Temple Franklin asking for letters of introduction for his travels through French cities. The following day two of Franklin's friends are known to have written to say they were sending him letters of introduction for Newenham.

These and other coincidences make it virtually certain that the unknown diarist was Sir Edward Newenham, and that an important Franklin correspondent, too little known to Franklin students because of the loss or dispersal of Newenham's papers, is now represented by an admirable diary which is pleasantly concerned with Franklin, and which has since been edited by Dixon Wecter (1941).

Franklin in 1735 printed a little book called "Cato's Moral Distichs," translated by James Logan, who was Franklin's patron. Of this translation of an ancient textbook, apparently of the fourth century, only a few copies are known to exist. One is in the Huntington Library. The Book Club of California in 1939 decided to bring out a facsimile of it. The book's chief interest lies in the fact that it is the earliest Latin classic both translated and printed in America—and that Franklin printed it. Franklin's preface has never been collected in any of his editions. I myself never read it till I was asked to write a brief introduction to the facsimile reprint. I should like to read you a part of Franklin's foreword, the Printer to the Reader. Already at twenty-nine he had the disarming candor and graceful knack at compliment which he always kept.

"In most Places that I am acquainted with, so great is the present Corruption of Manners, that a Printer shall find much more Profit in such Things as flatter and encourage Vice, than in such as tend to promote its contrary. It would be thought a Piece of Hypocrisy and pharisaical Ostentation in me, if I should say, that I print these Distichs more with a View to the Good of others than my own private Advantage: And indeed I cannot say it; for I confess, I have so great Confidence in the common Virtue and Good Sense of the People of this and the neighbouring Provinces, that I expect to sell a very good Impression."

Further matters that need to be investigated for the sake of Franklin's biography are: the intellectual and cultural history of Philadelphia during Franklin's rising years (at last

fully studied in Carl and Jessica Bridenbaugh's "Rebels and Gentlemen," 1942); the files of the *Pennsylvania Gazette*, which the Historical Society of Pennsylvania has reproduced in microfilm; Franklin's relations with the Quakers and with the Proprietarial government; Franklin's career as American lobbyist in England, in the new light of Professor Crane's discoveries. More should be known about Franklin's minor writings in France, particularly about his bagatelles. A really good edition of the bagatelles would be as valuable as delightful. His administration as President of Pennsylvania should be more exactly studied. There should be a reexamination of what had been done by his predecessors in various departments of science, and a detailed history of his reputation. His writings on electricity have at last been fully edited, and masterfully, by J. Bernard Cohen, in "Benjamin Franklin's Experiments" (1941).

Mr. Kenneth McKee in 1940 read at an American Philosophical Society meeting a paper on Franklin on the French stage in revolutionary Paris. Just after Franklin's death he appeared repeatedly as a character on the French stage, a hero passionately desirous of the rights of men and the rising cause of freedom.

Dr. Simon Flexner recently told me about his signing the Guest Book of the Royal Society. He asked to see Newton's autograph. So many before him had done this, and run their fingers beneath it as they read, that the name next after Newton's is completely wiped out. Dr. Flexner then turned to Franklin's name and found the name underneath his also worn away, though not quite so completely as the one under Newton's. Newton and Franklin were the only names so eminent as to have caused this obliteration of their neighbors.

But it must not be forgotten that Franklin, while a great scientist, was a great man of letters. He is no more to be judged merely by his "Autobiography" than Dr. Johnson is to be judged merely by "Rasselas" or the "Lives of the Poets." These men were greater writers than their books indicate. Their writings run through their lives.

Franklin said: "Prose writing has been of great value to me in my life, and was a principal means of my advancement." But he began by writing verse. His first Philadelphia friends,

obscure youths who talked literature and philosophy as they walked along the Schuylkill with him, were all poets. It was with a poet, James Ralph, that Franklin first went to England. Franklin thought that writing verse was the best way of learning to write prose, as it was. At twenty, when he sat down to write a scheme for the future conduct of his life, he had poetry in mind. He said: "Those who write of the art of poetry teach us that if we would write what may be worth reading we ought always, before we begin, to form a regular plan and design of our piece; otherwise we shall be in danger of incongruity. I am apt to think it is the same with life."

Thirty years later, writing to George Whitefield, Franklin used another poetical illustration: "Life, like a dramatic piece, should not only be conducted with regularity but methinks it should finish handsomely. Being now in the last act, I begin to cast about for something fit to end with. Or if mine be more properly compared with an epigram, as some of the lines are barely tolerable I am very desirous of concluding with a bright point."

Franklin in 1744 read Thomson's "Seasons" with tears of pleasure. In 1782 he read Cowper's poems, some of them twice over, "though," he said, "the relish for reading of poetry had long left me." But Franklin's drinking songs are probably his best expressions in verse.

In his prose it is obvious that he went through a great deal of experimentation in different modes. Though I have now too little time for a full discussion of this I can illustrate it with certain passages which show his range of prose style.

For instance, here is a sharp realism in his "Reflections on Courtship and Marriage":

"Let us survey the morning dress of some women. Downstairs they come, pulling up their ungartered, dirty stockings; slipshod, with naked heels peeping out; no stays or other decent conveniency, but all flip-flop; a sort of a clout thrown about the neck, half on and half off, with the frowzy hair hanging in sweaty ringlets, staring like Medusa with her serpents; shrugging up her petticoats, that are sweeping the ground and scarce tied on; hands unwashed, teeth furred, and eyes crusted —but I beg your pardon, I'll go no farther with this sluttish picture, which I am afraid has already turned your stomach."

If Franklin had put his mind to it he could have written as good novels as Fielding.

Here is the opening sentence of "Some Account of the Pennsylvania Hospital." Franklin, who had assisted Dr. Thomas Bond in the founding of the Hospital, as you know, was asked to write a history of it, which he published in 1754. His opening sentence is an example of homespun splendor hardly to be matched in the English language. The great sentences with which writers begin books commonly make use of flaming words; Franklin's words are all plain. His magic comes from his cadence and the emotion it implies.

"About the end of the year 1750 some persons who had frequent opportunities of observing the distress of such distempered poor as from time to time came to Philadelphia for the advice and assistance of the physicians and surgeons of that city; how difficult it was for them to procure suitable lodgings and other conveniences proper for their respective cases and how expensive the providing good and careful nurses and other attendants for want whereof many must suffer greatly, and some probably perish, that might otherwise have been restored to health and comfort and become useful to themselves, their families, and the public for many years after; and considering moreover that even the poor inhabitants of this city though they had homes were therein but badly accommodated in sickness and could not so well and easily be taken care of in their separate habitations as they might be in one convenient house, under one inspection and in the hands of skillful practitioners; and several of the inhabitants of the province who unhappily became disordered in their senses wandered about to the terror of their neighbors, there being no place (except the house of correction) in which they might be confined and subjected to proper management for their recovery, and that house was by no means fitted for such purposes; did charitably consult together and confer with their friends and acquaintances on the best means of relieving the distressed under those circumstances."

Franklin was perhaps most expert in his letters, always skillfully adapted to the circumstances or to the correspondents. Here is his grand style, as written to Washington in

1780, inviting Washington to come to Europe where Franklin was then minister to France:

"Should peace arrive after another campaign or two, and afford us a little leisure, I should be happy to see your Excellency in Europe and to accompany you, if my age and strength would permit, in visiting some of its ancient and most famous kingdoms. You would, at this side of the sea, enjoy the great reputation you have acquired, pure and free from those little shades that the jealousy of a man's countrymen and contemporaries are ever endeavoring to cast over living merit. Here you would enjoy, and know, what posterity will say of Washington. For a thousand leagues have nearly the same effect as a thousand years; the feeble voice of those grovelling passions cannot extend so far in either time or distance. At present I enjoy that pleasure for you, as I frequently hear the old generals of this martial country (who study the maps of America and mark upon them all your operations) speak with sincere approbation and great applause of your conduct; and join in giving you the character of one of the great captains of the age.

"I must soon quit the scene, but you may live to see our country flourish, as it will amazingly and rapidly after the war is over; like a field of young Indian corn, which long fair weather and sunshine had enfeebled and discolored, and which, in that weak state, by a thunder-gust of violent wind, hail, and rain seemed to be threatened with absolute destruction; yet the storm being past, it recovers fresh verdure, shoots up with double vigor, and delights the eye not of its owner only but of every observing traveler."

Franklin's impish style appears in his famous model for a letter of introduction. He wrote it in Paris where he was asked for so many letters of introduction for people that he did not know. I doubt that he ever used this for an actual letter—but only thought he would like to. This is what he wrote:

"The bearer of this, who is going to America, presses me to give him a letter of introduction, though I know nothing of him, not even his name. This may seem extraordinary, but I assure you it is not uncommon here. Sometimes, indeed, one unknown person brings another, equally unknown, to recom-

mend him; and sometimes they recommend one another. As to this gentleman, I must refer you to himself for his character and merits, with which he is certainly better acquainted than I can possibly be. I recommend him, however, to those civilities which every stranger of whom one knows no harm has a right to expect; and I request that you will do him all the good offices, and show him all the favor, that on further acquaintance you shall find him to deserve."

For Franklin letters were long-distance conversation. He wrote hundreds not only perfectly expressive of his own meanings but also delicately adapted to the recipients, whether children, elegant ladies, scientists, or politicians. Letters to him were a form of art. And they might be a relief, as in the famous letter to Strahan 5 July 1775, which Franklin wrote as a poet might have written a poem: to clear his mind and heart.

Franklin's method in writing was described by him in the last year of his life, in a letter of advice to Benjamin Vaughan:

"Before you sit down to write on any subject . . . spend some days in considering it, putting down at the same time, in short hints, every thought which occurs to you as proper to make a part of your intended piece. When you have thus obtained a collection of the thoughts, examine them carefully with this view, to find which of them is properest to be presented first to the mind of the reader, that he, being possessed of that, may the more easily understand it, and be disposed to receive what you intend for the second; and thus I would have you put a figure before each thought, to mark its future place in your composition. For so, every preceding proposition preparing the mind for that which is to follow, and the reader often anticipating it, he proceeds with ease, and pleasure, and approbation, as seeming continually to meet with his own thoughts. In this mode you have better chance for a perfect production; because, the mind attending first to the sentiments alone, next to the method alone, each part is likely to be better performed, and I think too in less time."

There are many revisions in his manuscripts. He wrote prose as careful poets write poetry. It is interesting to note certain changes he made in the text of his last speech before the Constitutional Convention, as revealed in the manuscript version in the Library of Cornell University. Franklin origi-

nally wrote: "I must own that there were several parts of this Constitution which I do not at present approve." He thought again, apparently, and certainly reduced the sentence of eighteen words to eleven: "I confess that I do not entirely approve of this Constitution." Or again, he wrote that he had often in his long life found himself obliged "to change opinions . . . which I once thought right but found to be wrong." The antithesis was too blunt, so he changed it to read: "which I once thought right but found to be otherwise." Cadence with Franklin was very important. In yet another passage he had spoken of the prejudices, passions, errors of opinion, local interests, and selfish views of any assembly of men. He first wrote: "From the fermentation of this heterogeneous mixture can a perfect production be expected?" He struck out "the fermentation of this heterogeneous mixture" and substituted simply "from such an assembly." The simplifying change reminds us of change in the Declaration where Jefferson had written: "We hold these truths to be sacred and undeniable." Franklin—for the writing seems clearly to be his—changed "sacred and undeniable" to "self-evident."

I need hardly speak of the "Autobiography" after Dr. Max Farrand's able discussion of it. But I should like again to call attention to the fact that Rousseau and Franklin together, at almost the same time, struck out all the essential lines the autobiography as a literary form was to take after them. Franklin's "Autobiography" is so familiar and so intimate that we hardly think of it as a book. It simply exists.

So with his masterly Poor Richard sayings. I have pointed out more than once that the economical maxims make up a smaller share of Poor Richard's wisdom than is commonly realized. They were collected partly for dramatic effect, in the long preface to the almanac for 1758 and were at once—and ever since—separately reprinted, whereas the others have been more or less buried in the rare original files. In a small volume in the inexpensive Pocket Books series, issued in 1940 on the 150th anniversary of Franklin's death, I selected, along with the "Autobiography" and minor writings, a hundred or so of the maxims of Poor Richard on the many topics he touched. (This selection has since then been translated into

Spanish and published in Mexico, the first volume in a new series of reprints initiated there in 1942.)

Here are some sayings of Poor Richard which must be taken into account if the full range of Franklin as aphorist is to be comprehended. "Thou hadst better eat salt with the philosophers of Greece than sugar with the courtiers of Italy." "The brave and the wise can both pity and excuse when cowards and fools show no mercy." "As charms are nonsense, nonsense is a charm." "Avarice and happiness never saw each other. How then should they become acquainted?" "Sloth and silence are a fool's virtues." "Hast thou virtue? Acquire also the graces and beauties of virtue." "If you would not be forgotten, as soon as you are dead and rotten, either write things worth reading, or do things worth the writing." "He that falls in love with himself will have no rivals." "Sin is not hurtful because it is forbidden, but it is forbidden because it is hurtful." "What's proper is becoming. See the blacksmith with his white silk apron." Others are:

"The most exquisite folly is made of wisdom spun too fine." "We are not so sensible of the greatest health as of the least sickness." "He that is secure is not safe." "He is not well bred that cannot bear ill breeding in others." "Having been poor is no shame, but being ashamed of it is." "Cunning proceeds from want of capacity." "'Tis against some men's principle to pay interest, and seems against others' interest to pay the principal." "If you would be loved, love and be lovable." "It is ill manners to silence a fool, and cruelty to let him go on." "Half a truth is often a great lie."

Of course Franklin drew his proverbs from many sources, but he almost always improved both the flavor and the cadence. For example, William Herbert in 1640 quoted: "Bells call others, but themselves enter not into the church." Franklin says: "The bell calls others to church, but itself never minds the sermon." Herbert said: "Help thyself, and God will help thee." Franklin: "God helps them that help themselves."

Franklin's sayings are likely to have graphic images and racy words. One of his sayings he tried three times. In 1740 he wrote "An empty bag cannot stand upright." In 1750: "An empty sack can hardly stand upright; but if it does, 'tis a

stout one." In 1758: "'Tis hard for an empty bag to stand upright"—the final form. A good instance of his way with an older proverb is to be found in what he did with the Scottish: "Fat housekeepers make lean executors." Franklin made it: "A fat kitchen, a lean will."

There is one saying first noted in 1572: "A gloved cat can catch no mice"; in 1611: "Cuffed cat's no good mouse-hunt"; in 1629: "A muzzled cat was never good mouser"; in 1641: "A gloved cat was never a good hunter"; in 1670: "A muffled cat is no good mouser." Then Franklin in 1754: "The cat in gloves catches no mice." To a sensitive ear the improvement in cadence is instant and irresistible.

Another one: Plautus in one of his plays said that no guest is welcome after three days. Lyly in "Euphues" made it: "Fish and guests in three days are stale." Sancho Panza in "Don Quixote" agreed with him. In 1670 the proverb appeared in an English collection as: "Fresh fish and new come guests, smell by they are three days old." A Scottish collection in 1721 gave it as: "Fresh fish and poor friends become soon ill sar'd"—ill savored. Then Franklin in 1736: "Fish and visitors stink in three days." The whole philosophy of the weekend.

Franklin's sayings are sometimes homely and rustic, as he meant them to be. His language belonged to the future because it had a simplicity and clarity that are timeless. If his sayings could travel backward in time instead of forward, they would probably seem as clear to his ancestors as they do to his descendants.

Franklin was first of all a great man, who was great in many ways. As he held few high offices, and made almost no speeches, he had to depend—and chose to depend—chiefly on writing as his means of making himself felt in the world. He had no Boswell to perpetuate his conversation. Franklin survives, directly or indirectly, in the written and remembered words he left behind him. The legendary Franklin comes, with some distortion, from the autobiographical Franklin, who was explicit and candid about himself, so far as he took the trouble to tell his story.

Parson Weems could turn Washington into a legend, but Weems, though he wrote a life of Franklin, could not compete

with Franklin's "Autobiography," the most widely read autobiography in the world. And in it and elsewhere Franklin is unique in English. No other writer in the language so perfectly combines "a felicitous elegance with a happy vernacular," the grace of art and the wit of nature. He had countless things to say, and never said one of them badly. A critic who thinks Franklin is not a great man of letters simply does not know what a man of letter is.

(Note: Revising this in August 1942, for publication, I have included some items of Franklin scholarship published since the address was originally delivered.)

Films of Thickness Tenth Length of Yellow Light Waves Found Best for Metal Details in Electron Microscope.—Plastic films one five hundred thousandth of an inch in thickness, equal to about a tenth of the length of yellow-orange light waves have been found best by General Electric scientists for revealing details of metal surfaces under the electron microscope. Such instruments magnify these details as much as 20,000 times, far greater than the useful limit of the ordinary optical microscope. These micrographs have already revealed important knowledge about metals used for war materials. A full account of the technique by which these films are made and used is published in a recent issue of the "Journal of Applied Physics," in an article by VINCENT J. SCHAEFER AND DR. DAVID HARKER, of the General Electric Research Laboratory. They perfected the method which is basically similar to one that Mr. Schaefer used as a hobby to preserve snowflakes and frost patterns. After the metal sample is polished and etched for a few seconds with acid, it is dipped into a dish containing a solution of Formvar, a plastic, in dioxane, a commercial solvent. The authors point out that it is necessary to do this within a few minutes after the sample has been etched and dried. Otherwise an infinitesimally thin film of grease or other contaminating material may start to form, and this affects the quality of the results. After dipping in the Formvar solution, the solvent evaporates, leaving the plastic film which is then stripped off. It reproduces the microscopic hills and valleys of the metallic crystals, and can be placed in the electron microscope for examination. Mr. Schaefer and Dr. Harker find that the exact thickness of the Formvar film is important, and this can be regulated by the strength of its solution in dioxane. Very thin films, about a millionth of an inch, do not show much contrast in the final electron picture. This seems to be because the film both on the top and bottom follows the hills and valleys of the metal. Thus it is all nearly equally transparent to the electron beam. If the film is as thick as 1/250,000 of an inch, contrast is also poor. Then the difference in the film between the hills and valleys is slight and all parts of the film offer nearly the same difficulty to the passage of the electrons which take the place of the light waves in the ordinary microscope. Best thickness is around a five hundred thousandth of an inch. These are just thick enough to make the top of the film level, while the bottom reproduces the hills and valleys. To the neglect of previous experimenters to appreciate these factors, the two G-E scientists attribute the failure of others to obtain satisfactory electron micrographs by films stripped directly from metal specimens.

R. H. O.

ON A GENERALIZATION OF KIRCHHOFF'S THEORY OF TRANSVERSAL PLATE VIBRATIONS IN THE VIBRATION PROBLEM OF STEAM TURBINE DISKS.

BY

I. MALKIN,

Boston, Mass.

(*Continued from October issue.*)

2. Remarks Concerning the Rigorous Generalization of Kirchhoff's Theory for the Idealized Disk with Blades of Any Length. —Before extending the results of the preceding section to vibrations with more than two nodal diameters and also to disks with longer blades (so as if the "wheel vibrations" of the disk were possible with blades of any length), we shall insert here a few remarks concerning a *rigorous* generalization of Kirchhoff's theory of transversal plate vibrations to the case of the idealized disk (Fig. 2), a generalization which holds true for disks with blades of *any* length.

The Hamiltonian Integral is represented in the rigorous treatment, just as in that of the preceding section, by a sum of two integrals, the first of which refers to the domain Γ and the second to the domain Γ^* (Fig. 2). The first of these two integrals corresponds to that occurring in the original theory of Kirchhoff. The analysis given in the preceding section is based upon the assumption that the lower limit of the first integral is $r = 0$, like in the original theory of Kirchhoff. In the case of a *combined* system of the kind represented in Fig. 2, we have to take the first integral between the limits $r = r_1 > 0$ and $r = a$, and to consider the lower limit r_1 as an additional parameter of the problem; at this lower limit r_1 of the first integral we will have two additional "enforced" boundary conditions:

$$\left.\begin{aligned} W &= 0 \\ \frac{\partial W}{\partial r} &= 0 \end{aligned}\right\} \quad \text{when } r = r_1 \left.\right\} \qquad (20)$$

$$(W = 0 \text{ when } 0 \leq r \leq r_1).$$

The (extended) Hamiltonian Integral must be made a minimum also with respect to the parameter r_1. This constitutes a well known problem of the Calculus of Variations,[17] of which the variational problem of Kirchhoff's theory represents obviously a special case only.

Starting from the analogy with the analysis, given in the preceding section, for the case of a *full* disk (i.e. a disk without resting core) with blades, we can now form an adequate idea about the procedure of soluiton of the new variational problem considering the following two points:

1. The general solution, fulfilling the additional boundary conditions (20), for the wheel part $r_1 \leq r \leq a$, without blades and free at the outer boundary $r = a$, is known [18] and can be used in the process of forming the frequency equations analogous to the above equations (19);

2. Having, as a result of that procedure, the solution of the system of frequency equations in terms of the parameter r_1, it will be necessary to solve a maximum-minimum problem with respect to the argument r_1 in order to obtain the minimum value for the fundamental (lowest) frequency.

Taking into account the complications introduced by the additional boundary conditions (20) [19] and judging by the tedious computations necessary even in the simple case of a full disk in the preceding section, we realize that a rigorous treatment of the problem of the actual ("disk blade") vibrations, even of the *idealized* disk, will require very cumbersome calculations which are to be avoided in practical conditions. Recourse must be taken, therefore, to approximate methods for the necessary frequency estimations (see section 7).

At the present moment we shall resume the preliminary frequency computations started with in section 1.

3. *Frequency Computation of the "Wheel Vibrations" of the Idealized Disk by Means of the Rayleigh Ritz Stodola Method.*— In applying the Rayleigh Ritz Stodola method of frequency

[17] See f.i. O. Bolza, "Variationsrechnung," Leipzig und Berlin, 1909, Ch. VI, esp. p. 314.

[18] See R. V. Southwell, *Proc. Roy. Soc. London* (A), 101, 133–153, (1922), furthermore K. Klotter, "Ingenieurarchiv," 1930, p. 126 ff.; the latter paper gives valuable additional information on the literature of the subject.

[19] See especially the paper of K. Klotter mentioned in the preceding footnote.

computation [20] to a disk with long blades the difficulty was considered to be caused by the question of properly inserting the potential energy of deformation of the blades (see Introduction, *d*); in a disk with short blades the potential energy of blade deformation was assumed to be zero. In the following system (21), (22) of formulæ for the hypothetical form of vibration of the disk with blades the vibration form of the latter is evidently general enough as to permit a sufficient amount of variation of the bending energy of the blade; it has four constants (A, B, C, p) to satisfy the four conditions involved. In other words, no assumptions concerning the vibration form of the blade will be necessary, because our procedure, characterized by a sufficient number of available constants, will yield that vibration form automatically.

In the analysis given in this section 3 the vibrating system is again represented by the idealized disk visualized by Fig. 2.

Designating by Ω the hypothetical frequency we shall have

$$W = f(r) \sin \vartheta \cos \Omega t \qquad \text{when } 0 \leq r \leq a,$$
$$W^* = f^*(r) \sin \vartheta \cos \Omega t \qquad \text{when } a \leq r \leq a + \delta a,$$

and we assume that the vibration forms $f(r)$ and $f^*(r)$ can be approximated by the expressions

$$f(r) = r^s. \tag{21}$$

$$f^*(r) = A + Br + Ce^{-p(r-a)}. \tag{22}$$

The constants occurring in (22) are to be determined from the conditions expressed by the equations (7), (8) and (3), (4) in section 1. These conditions lead here to the equations

$$A + Ba + C = a^s, \tag{23}$$

$$Ba - Cpa = sa^s, \tag{24}$$

$$C(pa)^2 = \frac{s(s-1) + \nu(s-k^2)}{f(1-\nu^2)}\, a^s, \tag{25}$$

$$C(pa)^3 = \frac{(1-\nu)k^2(s-1) - (s^2-k^2)(s-2)}{f(1-\nu^2)}\, a^s, \tag{26}$$

[20] See A. Stodola, "Schweizerische Bauzeitung," 1914, or A. Stodola, "Steam and Gas Turbines," New York, 1927, section 187.

from which the constants A, Ba, C and pa are to be computed in terms of the parameter s. The results of such a computation, carried through (with logarithms to 5 decimal places) for $k = 2$, $l = 0$, and $f = 1$, are given in the Table 2, in which

TABLE 2.
Solution of the System of Equations (23)–(26).

s.	pa.	$\bar{A}$.	$\bar{Ba}$.	$\bar{C}$.
1.6	4.600	−0.669798	1.6573338	0.012464
1.7	3.254	−0.920745	1.8688538	0.051891
1.8	2.677	−1.239802	2.1201923	0.11961
1.9	2.297	−1.641502	2.4166222	0.22488
2.0	2.000	−2.153853	2.7692333	0.38462
2.1	1.747	−2.821625	3.194795	0.62683
2.2	1.520	−3.717091	3.718241	0.99885

the upper dash designates division by a^s, so that $\bar{A} = Aa^{-s}$, and so forth.

Having thus determined the constants in terms of the parameter s, we compute the energy amounts of the two parts of the idealized system represented in Fig. 2, i.e. of the solid circular plate of constant thickness (domain Γ) and of the "blades" (domain Γ^*).

The energy amounts of the wheel of constant thickness are given by the following formulæ (see below section 5):

$$\text{Pot. En.} = U_0 \cos^2 \Omega t = \frac{\pi E h^3}{1 - \nu^2} \frac{1}{6s - 6} a^{2s-2} S \cos^2 \Omega t, \quad (27)$$

wherein

$$S = (s^2 - k^2)^2 + 2(1 - \nu)(s - 1)[k^2(2s - 1) - s^2]$$

$$\text{Kin. En.} = \Omega^2 L_0 \sin^2 \Omega t = \Omega^2 \pi \rho h a^{2s+2} \frac{1}{2s + 2} \sin^2 \Omega t. \quad (28)$$

The corresponding energy amounts of the blade system are

$$\text{Pot. En.} = U_0^* \cos^2 \Omega t = \frac{1}{3} E a h^3 f \int_{r=a}^{a+\delta a} \int_{\vartheta=0}^{2\pi} (W_{rr}^*)^2 dr d\vartheta$$

$$= \frac{\pi}{6} \frac{E h^3 f}{a^2} C^2 (pa)^3 (1 - e^{-2p\delta a}) \cos^2 \Omega t, \quad (29)$$

$$\text{Kin. En.} = \Omega^2 L_0^* \sin^2 \Omega t = ha\rho f \int_{r=a}^{a+\delta a} \int_{\vartheta=0}^{2\pi} (W_t^*)^2 dr d\vartheta$$

$$= \Omega^2 \pi h a^2 \rho f \{s\} a^{2s} \sin^2 \Omega t, \quad (30)$$

wherein

$$\left.\begin{aligned}\{s\} = {} & \bar{A}^2\left(\frac{a'}{a}-1\right)+\frac{1}{3}(\bar{B}a)^2\left[\left(\frac{a'}{a}\right)^3-1\right] \\ & +\bar{A}\bar{B}a\left[\left(\frac{a'}{a}\right)^2-1\right]+\frac{\bar{C}^2}{2pa}(1-e^{-2p\bar{s}a}) \\ & +\frac{2\bar{A}\bar{C}}{pa}(1-e^{-p\bar{s}a})+\frac{2\bar{B}a\bar{C}}{pa}\left[1+\frac{1}{pa}\right. \\ & \left.-\left(\frac{a'}{a}+\frac{1}{pa}\right)e^{-p\bar{s}a}\right].\end{aligned}\right\} \tag{31}$$

An approximate value of the frequency follows now from the relation

$$\Omega^2 \leqq \left[\frac{U_0(s)+U_0^*(s)}{L_0(s)+L_0^*(s)}\right]_{\min.} \tag{32}$$

Introducing the values, given in Table 2, into the expressions (29), (30) and substituting the latter as well as the expressions

TABLE 3.

The Values of the Functions S, $(1-\nu^2)[s]$, $(s+1)^{-1}$, $2\{s\}$ *and* Φ.

s.	1.6.	1.7.	1.8.	1.9.	2.0.	2.1.	2.2.
S	12.19200	11.15414	10.74600	10.79300	11.2000	11.89882	12.85200
$(s+1)^{-1}$	0.38462	0.37038	0.35714	0.34483	0.33333	0.32258	0.31250
$(1-\nu^2)[s]_{\mathrm{I}}$	0.01289	0.07244	0.19964	0.41732	0.75256	1.23711	1.90748
$(1-\nu^2)[s]_{\mathrm{II}}$	0.01354	0.07991	0.22730	0.48734	0.89892	1.50952	2.37650
$(1-\nu^2)[s]_{\mathrm{III}}$	0.01371	0.08273	0.23969	0.52250	0.97924	1.67081	2.67375
$2\{s\}_{\mathrm{I}}$	0.93880	0.96894	1.00038	1.03286	1.06702	1.10284	1.13996
$2\{s\}_{\mathrm{II}}$	1.71878	1.80102	1.88816	1.98038	2.07754	2.18180	2.28784
$2\{s\}_{\mathrm{III}}$	2.75736	2.93148	3.12006	3.32348	3.54322	3.78152	4.03884
Φ_{I}	9.22225	8.38240	8.06283	8.13860	8.53540	9.21550	10.16163
Φ_{II}	5.80286	5.17367	4.88722	4.85222	5.01844	5.35400	5.85638
Φ_{III}	3.88464	3.40323	3.15936	3.08521	3.14171	3.30368	3.56808

(27), (28) into the formula (32), we find

$$\Omega^2 \leqq \frac{Eh^2}{3(1-\nu^2)\rho a^4}\left[\frac{S+(1-\nu^2)[s]}{(s+1)^{-1}+2\{s\}}\right]_{\min.},$$

where

$$[s] = \bar{C}^2(pa)^3(1-e^{-2p\bar{s}a}), \tag{32a}$$

while $\{s\}$ is given by the formula (31). With the constants obtained in Table 2 the quantities S, $(1-\nu^2)[s]$, $(s+1)^{-1}$ and $2\{s\}$ assume, for $k=2$, the system of values given in

Table 3. The items $(1-\nu^2)[s]$ and $2\{s\}$, referring to the blades, are calculated in Table 3 for the three blade lengths

$$\text{I. } \delta a = 0.30a, \qquad \text{II. } \delta a = 0.45a, \qquad \text{III. } \delta a = 0.60a. \tag{33}$$

The Table 3 gives also the corresponding values of the frequency function

$$\Phi = \frac{S + (1-\nu^2)[s]}{(s+1)^{-1} + 2\{s\}}.$$

Plotting now the function Φ of the Table 3 as a function of s for the three blade lengths (33) we obtain the three curves of Fig. 6. The minimum ordinates Φ_{I}, Φ_{II}, Φ_{III} of these curves

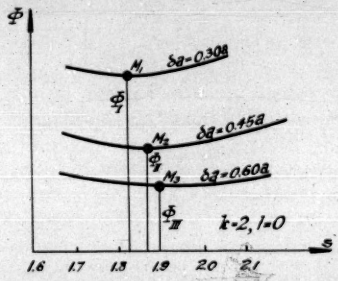


FIG. 6.

determine the frequencies, and so we find

$$\Omega_{\text{I}}^2 \leqq \frac{Eh^2}{3(1-\nu^2)\rho a^4}\,8.06; \qquad \Omega_{\text{II}}^2 \leqq \frac{Eh^2}{3(1-\nu^2)\rho a^4}\,4.85;$$

$$\Omega_{\text{III}}^2 \leqq \frac{Eh^2}{3(1-\nu^2)\rho a^4}\,3.09. \tag{33a}$$

It is interesting to observe that the abscissæ s_{I}, s_{II}, s_{III} of the minima increase with increasing blade length, quite in accordance with the qualitative estimation, suggested by physical experience, that the curvature of the vibrating disk must be larger in the case of larger blade masses, while the frequency is getting lower at the same time.

The conclusion to be drawn from the results obtained in Table 3 is based upon the fact that the values $(1-\nu^2)[s]$ are

almost negligible, irrespectively of the blade length, as compared with the corresponding values of the quantity S; this means that the potential energy of the blades is of little effect on the vibration frequency of the entire system, all the more the frequency is proportional to the square root of $\Phi_{min.}$. This confirms and extends the result, obtained in section 1, that the vibration form of the blade is not essentially different from a straight line. The vibration form of the blade can be easily calculated directly from the Tables 2 and 3 by introducing the corresponding values of A, B, C, p into the formula (22).

Comparing the results with those of section 1 we furthermore find that, while the exact method of section 1 yields the frequency

$$\frac{\alpha^2}{a^2}(\lambda a)^2 = \sqrt{\frac{Eh^2}{3(1-\nu^2)\rho a^4}}\,2.89$$

(see Table 2) for the blade length $\delta a = 0.288a$, with $k = 2$, $l = 0$, the Rayleigh Stodola method gives us, in the case of the somewhat larger blade length $\delta a = 0.300a$, with the same nodal figure, the somewhat lower frequency as represented by the factor $\sqrt{8.06} = 2.84$; the agreement of both methods is thus very satisfactory.

The results expressed in the last but one paragraph (of this section) hold true for $k \geqslant 3$ as well; it was not deemed necessary, however, to reproduce here the numerical solution as it has been done in the case of $k = 2$ in Table 3.

The frequency curve as a function of the blade length, as obtained, for $k = 2$ and $l = 0$, in (33a), is shown again in Fig. 5, right part. It is interesting to note the continuous connection of both branches of the frequency curve obtained from the two different methods.

4. Concluding Remarks to Sections 1–3 and Preparatory Data for Application of the Approximate (Modified) Procedure to a Practical Example.—As already mentioned above in advance the results obtained in sections 1 and 3 are in a very good mutual agreement. We have furthermore, in particular, already called attention to the circumstance that the vibration form of the blade in the "wheel vibrations" of the combined system is represented, with practically sufficient accuracy, by

a straight line, as *assumed* in Stodola's original computing procedure. This result rehabilitates, in other words, Stodola's calculation of the "wheel vibration" frequencies, as far as such vibrations are possible.

Starting, however, from the observation that the "wheel vibrations" represent only an extreme case of the *actual* ("disk blade") vibrations, we are led to the conclusion that, if the latter are to be determined, a modification of the analytical method used above must take place. As the exact form of the modified problem would cause rather considerable practical difficulties, we shall substitute subsequently an approximate solution for the rigorous method, along the general lines drawn up in section *f* of the Introduction.

In order to obtain an illustration as close as possible to the actual conditions of steam turbine design, we shall apply the approximate solution to a conical steam turbine disk, with blades of some varying but otherwise practically possible dimensions. As to the dimensions of the disk we chose, with reference to the designations of Fig. 7, the values $R = 31.0$ in.; $a = 21.7$ in.; $r_0 = 3.15$ in.; $H = 0.80$ in.; $h_a = 0.236$ in. The blade bucket may be determined by the following data (see Stodola's handbook, section 70, table at the end of part 1):

Cross section area Q^* of the blade equal to $0.588 \times 2h_a \times b$;

Moment of inertia J^* of this cross section area equal to $0.0278 \times (2h_a)^3 \times b$ (the width b of the cross section will not appear in the final formulæ);

Blade length varying between the limits 0 and $0.6a$;

The coefficient $f = n^*Q^*: (2\pi a \times 2h_a) = 1/3$.

The general expressions for the energy amounts of the conical disk will be derived and tabulated in section 5; the corresponding formulæ for the blades are given in section 6; the results will be then used in section 7 for an approximate determination of the frequencies of the "disk blade" vibrations.

5. *Computation of the Energy Amounts for the Conical Disk.*—The general expressions for the energy amounts of the disk wheel of any profile (meridian curve) are,[21] with $f(r)$ given

[21] See f. i. A. Stodola, "Steam and Gas Turbines," New York, 1927, section 187.

by (21), [cf. (27), (28)],

$$\Omega^2 L_0 = L_{(\sin^2 \Omega t - 1)} = \Omega^2 \rho \pi \int_{r_0}^{a} [f(r)]^2 h r dr, \tag{34}$$

$$U_0 = U_{(\cos^2 \Omega t - 1)} = \frac{\pi E}{3(1 - \nu^2)} \times \int_{r_0}^{a} \left\{ \left(\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - k^2 \frac{f}{r^2} \right)^2 - 2(1 - \nu) \times \left[\frac{d^2 f}{dr^2} \left(\frac{1}{r}\frac{df}{dr} - k^2 \frac{f}{r^2} \right) - k^2 \left[\frac{d}{dr}\left(\frac{f}{r} \right) \right]^2 \right] \right\} h^3 r dr, \tag{35}$$

where $h = h(r)$ designates the variable half thickness of the

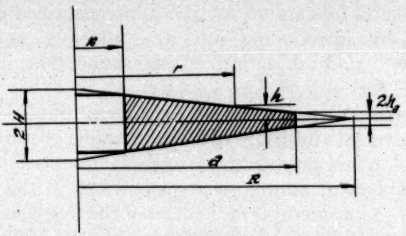


FIG. 7.

wheel. In the case of a conical disk we will have, with the notations of Fig. 7,

$$h = H\left(1 - \frac{r}{R}\right) \qquad (H = \text{const.};\ R = \text{const.}). \tag{36}$$

Substituting, in the case of the "wheel vibrations" of the conical disk, with $k = 2, 3, 4, \cdots$ nodal diameters, but with no nodal circles, the formulæ (21), (36) into the expressions (34), (35) we finally find

$$\left.\begin{aligned} \Omega^2 L_0 &= \Omega^2 \pi \rho H R^{2s+2} F_1(\chi, s)\Big|_{x_1}^{x_2}, \\ U_0 &= \frac{\pi E H^3}{1 - \nu^2} R^{2s-2} F_2(\chi, s, k)\Big|_{x_1}^{x_2}, \end{aligned}\right\} \tag{37}$$

wherein

$$\left.\begin{aligned} \chi_1 = r_0/R, \quad \chi_2 = a/R, \\ F_1(\chi, s) = \chi^{2s+2}\left(\frac{1}{2s+2} - \frac{\chi}{2s+3}\right), \end{aligned}\right\} \tag{38}$$

$$\begin{aligned} F_2(\chi, s, k) = \chi^{2s-2}[&(s^2 - k^2)^2 \\ &+ 2(1-\nu)(s-1)\{k^2(2s-1) - s^2\}] \\ &\times \left(\frac{1}{6s-6} - \frac{\chi}{2s-1} + \frac{\chi^2}{2s} - \frac{\chi^3}{6s+3}\right). \end{aligned} \tag{39}$$

The functions $F_1(\chi, s)$ and $F_2(\chi, s, k)$ are tabulated in the following Standard Tables I and II.[22]

For a wheel of constant thickness the formulæ for the energy amounts (see above, section 3) are obtained from (37), (38), (39) by introducing into the latter $\chi_1 = r_0/R = 0$, $\chi_2 = a/R = 0$, and taking into consideration that, in the case in question, $R\chi_2 = a \neq 0$.

6. Frequency Computation for Transversal Blade Vibrations.—The transversal vibrations of a blade, or of a group of blades, with the disk wheel assumed to be entirely at rest, have equally been the subject of numerous investigations in the technical literature.[23] Considerable difficulties of the frequency computation are caused by the circumstance that the blades are not rigidly fastened in the wheel rim.

For our present purposes the frequency computation of the blades can be carried out as follows.

In the ideal case, when the blade is rigidly fastened in the rim of the disk (and if the participation of the disk wheel in the vibration of the blade is entirely prevented), the vibration problem of the blade is analytically expressed by the differential equation (2) in connection with the boundary conditions

$$\left.\begin{aligned} f^*(r) = 0 \\ df^*(r)/dr = 0 \end{aligned}\right\} \text{ when } r^* = 0, \tag{40, 41}$$

[22] Following a suggestion by the author, the Standard Tables I and II have been computed by the special calculation group of the Westinghouse Electric & Manufacturing Company in South Philadelphia, Pa., some years ago.

[23] See f.i. the paper of K. Karas in the "Ingenieurarchiv," Berlin, 1934, pp. 325–352, giving also valuable information on the literature of the subject.

Table I.

Values of $F_1(x, s)$.

Column.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
$x \rightarrow$	0.10.	0.15.	0.20.	0.25.	0.30.	0.35.	0.40.	0.50.	0.60.	0.70.	0.80.	0.90.
$s \downarrow$												
1.00	.000023	.000111	.000336	.000781	.001539	.002701	.004352	.009375	.016848	.026411	.036864	.045927
1.25	.000006	.000038	.000133	.000345	.000744	.001408	.002420	.005803	.011174	.019074	.028124	.036466
1.50	.000002	.000013	.000053	.000155	.000363	.000744	.001603	.002604	.007776	.014006	.021845	.029525
1.75	.000001	.000005	.000022	.000070	.000181	.000398	.000779	.002318	.005391	.009901	.017216	.024288
2.00		.000002	.000009	.000032	.000090	.000214	.000449	.001488	.003777	.007843	.013731	.020245
2.25		.000001	.000004	.000015	.000045	.000117	.000260	.000963	.002669	.005956	.011062	.017064
2.50			.000002	.000007	.000023	.000064	.000152	.000628	.001900	.004559	.008988	.014520
3.00				.000001	.000006	.000019	.000053	.000275	.000980	.002722	.006058	.010762
3.50					.000002	.000006	.000019	.000119	.000515	.001614	.004176	.008179
4.00						.000002	.000007	.000053	.000275	.001027	.002928	.006340
4.50						.000001	.000002	.000024	.000148	.000624	.002082	.004992
5.00							.000001	.000011	.000081	.000408	.001498	.003983
5.50								.000005	.000044	.000261	.001087	.003212
6.00								.000002	.000025	.000168	.000796	.002614
6.50								.000001	.000014	.000109	.000586	.002145
7.00									.000008	.000071	.000435	.001771

TABLE II.

Values of $F_2(x, s)$.

$K = 2$.

$x \rightarrow$	0.10.	0.15.	0.20.	0.25	0.30.	0.35.	0.40.	0.50.	0.60.	0.70.	0.80.	0.90.
$s \downarrow$												
1.00												
1.25	1.4312	1.6700	1.8386	1.9617	2.0529	2.1205	2.1705	2.2333	2.2646	2.2782	2.2830	2.2840
1.50	.2031	.2823	.3487	.4037	.4488	.4852	.5141	.5537	.5755	.5858	.5897	.5906
1.75	.0478	.0799	.1118	.1418	.1689	.1928	.2131	.2436	.2621	.2716	.2754	.2764
2.00	.0152	.0318	.0490	.0685	.0881	.1067	.1238	.1517	.1704	.1809	.1854	.1866
2.25	.0057	.0139	.0254	.0392	.0545	.0704	.0861	.1140	.1347	.1474	.1531	.1547
2.50	.0022	.0067	.0140	.0240	.0363	.0501	.0646	.0932	.1165	.1319	.1395	.1417
3.00	.0004	.0016	.0044	.0102	.0167	.0266	.0385	.0664	.0944	.1164	.1284	.1325
3.50		.0004	.0014	.0035	.0075	.0138	.0226	.0472	.0772	.1048	.1227	.1293
4.00			.0004	.0013	.0033	.0069	.0128	.0329	.0621	.0938	.1176	.1275
4.50				.0004	.0014	.0033	.0070	.0219	.0488	.0829	.1122	.1260
5.00					.0005	.0016	.0037	.0144	.0376	.0723	.1065	.1246
5.50						.0008	.0019	.0092	.0285	.0622	.1004	.1231
6.00							.0010	.0058	.0212	.0529	.0941	.1216
6.50							.0005	.0036	.0156	.0446	.0876	.1200
7.00							.0002	.0022	.0113	.0372	.0811	.1182

TABLE II.—*Continued.*

$K = 3$.

$x \rightarrow$	0.10.	0.15.	0.20.	0.25.	0.30.	0.35.	0.40.	0.50.	0.60.	0.70.	0.80.	0.90.
$s \downarrow$												
1.00												
1.25	11.3618	13.2571	14.5957	15.5731	16.2964	16.8332	17.2299	17.7291	17.9773	18.0855	18.1230	18.1312
1.50	1.6217	2.2540	2.7841	3.2236	3.5834	3.8739	4.1045	4.4209	4.5949	4.6774	4.7081	4.7152
1.75	.3257	.5446	.7620	.9666	1.1519	1.3145	1.4532	1.6608	1.7871	1.8522	1.8781	1.8845
2.00	.0777	.1623	.2504	.3501	.4498	.5449	.6321	.7746	.8704	.9240	.9469	.9529
2.25	.0209	.0513	.0935	.1445	.2009	.2594	.3171	.4199	.4963	.5428	.5641	.5700
2.50	.0062	.0184	.0385	.0659	.0995	.1373	.1773	.2553	.3194	.3617	.3825	.3886
3.00	.0007	.0029	.0080	.0185	.0302	.0480	.0696	.1200	.1705	.2103	.2319	.2393
3.50		.0005	.0019	.0051	.0108	.0198	.0324	.0678	.1108	.1503	.1760	.1856
4.00			.0005	.0016	.0041	.0087	.0162	.0476	.0786	.1189	.1490	.1616
4.50				.0005	.0016	.0039	.0083	.0259	.0577	.0980	.1326	.1489
5.00					.0006	.0018	.0042	.0163	.0426	.0819	.1207	.1412
5.50						.0008	.0021	.0102	.0314	.0686	.1108	.1358
6.00							.0011	.0063	.0230	.0574	.1019	.1317
6.50							.0005	.0038	.0167	.0477	.0937	.1283
7.00							.0002	.0023	.0119	.0393	.0859	.1252

TABLE II.—*Continued.*

$K = 4$.

$x \rightarrow$	0.10.	0.15.	0.20.	0.25.	0.30.	0.35.	0.40.	0.50.	0.60.	0.70.	0.80.	0.90.
$s \downarrow$												
1.00												
1.25	41.3061	48.1970	53.0632	56.6138	59.2460	61.1976	62.6392	64.4547	65.3570	65.7506	65.8869	65.9165
1.50	6.0150	8.3604	10.3265	11.9565	13.2911	14.3684	15.2238	16.3975	17.0429	17.3490	17.4626	17.4889
1.75	1.2061	2.0169	2.8224	3.5801	4.2662	4.8685	5.3822	6.1513	6.6384	6.8599	6.9560	6.9795
2.00	.2791	.5832	.9001	1.2582	1.6166	1.9586	2.2720	2.7842	3.1284	3.3212	3.4036	3.4251
2.25	.0706	.1733	.3159	.4883	.6790	.8769	1.0716	1.4192	1.6774	1.8345	1.9066	1.9265
2.50	.0190	.0569	.1188	.2036	.3072	.4242	.5477	.7887	.9865	1.1172	1.1815	1.2003
3.00	.0016	.0071	.0197	.0455	.0744	.1179	.1710	.2950	.4190	.5171	.5703	.5884
3.50		.0010	.0038	.0100	.0212	.0388	.0635	.1328	.2173	.2947	.3451	.3639
4.00			.0008	.0027	.0067	.0143	.0265	.0683	.1287	.1946	.2438	.2644
4.50				.0008	.0023	.0056	.0119	.0372	.0828	.1408	.1905	.2139
5.00					.0008	.0023	.0055	.0214	.0559	.1075	.1583	.1852
5.50						.0010	.0026	.0125	.0387	.0846	.1365	.1674
6.00							.0013	.0074	.0271	.0676	.1202	.1553
6.50							.0006	.0044	.0191	.0545	.1071	.1465
7.00							.0002	.0026	.0134	.0439	.0959	.1398

$$\left.\begin{aligned} d^2f^*(r)/dr^2 &= 0 \\ d^3f^*(r)/dr^3 &= 0 \end{aligned}\right\} \text{when } r^* = \delta a, \qquad (42), (43)$$

wherein $r^* = r - a$. The general integral of (2) is represented by (11). Substituting into the latter the boundary conditions (40)–(43), we find the well known frequency equation

$$1 + \cos \lambda^* \delta a \cosh \lambda^* \delta a = 0. \qquad (44)$$

The smallest root of this equation being equal to 1.875, the lowest vibration frequency of the blade will be

$$\Omega = \alpha^{*2}\lambda^{*2} = \sqrt{\frac{E}{\rho}\frac{J^*}{Q^*}}\left(\frac{1.875}{\delta a}\right)^2 \qquad (45)$$

If the blade is not rigidly fastened in the rim, the vibration frequency of the blade will be lower than represented by (45). To express the actual conditions more accurately, the condition (41) may be replaced by [24]

$$\frac{df^*(r)}{dr} = \beta \frac{d^2f^*(r)}{dr} \qquad \text{when} \qquad r^* = 0, \qquad (46)$$

wherein β is a constant which must be determined experimentally. With the boundary conditions (40), (42), (43), and (46) the frequency equation assumes the form

$$\beta\lambda^*(\sin \lambda^*\delta a \cosh \lambda^*\delta a - \cos \lambda^*\delta a \sinh \lambda^*\delta a) = 1 + \cos \lambda^*\delta a \cosh \lambda^*\delta a \qquad (47)$$

used for the curve Ω^* in Fig. 3, with a special value of β, given below.

7. *Asymptotic Representation of the Actual ("Disk Blade") Vibration Frequencies.*—In using the formulas and tables given in the last two sections we are now able to offer a simple procedure for an approximate determination of the frequencies of the "disk blade" vibrations of the steam turbine disk. This procedure, subsequently applied to the example chosen in section 4, can be characterized, according to the plan indicated already above, shortly as follows: we shall determine first the frequencies of the "wheel vibrations," then the frequencies of the blade vibrating alone (i.e. with the disk entirely at rest),

[24] See W. Lohmann, "Eigenschwingungen von Dampfturbinenschaufeln," Jena, 1932, p. 63.

and these two preparatory computations will form the basis for the approximate determination of the "disk blade" vibration frequencies, by yielding certain limiting lines for the frequency curves.

A. Wheel Vibrations.—The frequency Ω_k of the "wheel vibration," with $k = 2, 3, 4, \cdots$ nodal diameters and without nodal circles, of the disk considered (see section 4), can be obtained by introducing into formula (32) the expressions U_0^*, L_0^*, given in (29), (30), and the expressions U_0, L_0, given in (37), (38), (39). Replacing h^3 in (29) by $3J^*/2b$ ("idealized blade" = bar of rectangular cross section $2h \times b$) and substituting then for J^* the value given above for the special blade considered (see section 4), we will have, with $f = 1/3$, in terms of the parameter s occurring in the formula (21),

$$\Omega_k^2 \leqq \frac{EH^2}{\rho(1-\nu^2)R^4} \times \left[\frac{F_2\Big|_{\chi_1}^{\chi_2} + 0.111\,\frac{1-\nu^2}{6}\left(\frac{h_a}{H}\right)^3\left(\frac{a}{R}\right)^{2s-2}[s]}{F_1\Big|_{\chi_1}^{\chi_2} + \frac{1}{3}\frac{h_a}{H}\left(\frac{a}{R}\right)^{2s+2}\{s\}} \right]_{\min.}, \quad (48)$$

wherein the expressions $F_j (j = 1, 2)$ are the functions (38), (39) given in the Standard Tables I and II, so that the quantities $F_j| = F_j(s, \chi_2) - F_j(s, \chi_1) = F_j(s, a/R) - F_j(s, r_0/R)$ can be easily computed; the expressions $(1-\nu^2)[s]$ and $\{s\}$, defined by the formulæ (32a) and (31) resp., are given numerically in Table 3 for $\delta a/a = 0.30; 0.45; 0.60$; these functions vanish for $\delta a/a = 0$. Introducing the functions calculated and the constants, given above, into (48), we obtain, after some additional computation, for $k = 2$, the results given in Table 4, in which Φ designates the expression within the square brackets in (48). Representing the minima of the function $\Phi^{1/2}$ graphically as a function of the blade length we obtain, for $k = 2$, the curve Ω_2 in Fig. 3. The curves $\Omega_3, \Omega_4, \cdots$ in the same figure are to be determined quite analogously. For the purpose of the present contribution, however, it is sufficient to determine the points only, in which the curves $\Omega_3, \Omega_4, \cdots$ intersect the axis of the ordinates ($\delta a = 0$, disk without blades), plotting the rest of these higher branches $\Omega_3, \Omega_4, \cdots$ of the frequency curve only approximately.

B. Blade Vibrations.—If the blades are vibrating alone, *with the disk wheel entirely at rest*, the vibration frequencies may be represented by the equation (47). The coefficient β vanishes in this equation, if the blades are rigidly fastened in the disk rim; otherwise $\beta > 0$. Assuming, as an approximation, the coefficient β to be equal to $0.15a$, we will find the smallest root of equation (47) to be given by

$$\lambda^*\delta a = 1.25 \qquad 1.42 \qquad 1.51 \qquad 1.57$$

when

$$\delta a = 0.15a \qquad 0.30a \qquad 0.45a \qquad 0.60a.$$

Substituting the values of $\lambda^*\delta a$, as well as those of J^* and Q^* given above, into the formula $\Omega^* = \alpha^{*2}\lambda^{*2}$ [cf. equations (12)

TABLE 4.

The Quantity Φ as a Function of s and δa for $k = 2$.

$\delta a/a \downarrow$.	$s \to 1.75$.	2.00.	2.25.	2.50.	3.00.
0.00	22.6	21.08	23.8	28.5	42.6
0.30	13.41	11.87	12.34	13.79	
0.45	9.86	8.36	8.24	8.67	
0.60	7.25	5.86	5.46	5.43	5.48

and (45)] we will obtain the blade frequency

$$\Omega^* = 3130 \qquad 1000 \qquad 500 \qquad 300$$

when

$$\delta a = \quad 0.15a \qquad 0.30a \qquad 0.45a \qquad 0.60a.$$

The blade frequency Ω^* is represented graphically in Fig. 3 as a function of the blade length.

C. Disk Blade Vibrations.—Having found the wheel vibrations, as represented by the curves Ω_k, $k = 2, 3, 4, \cdots$, in Fig. 3, and the blade vibrations, as represented by the curve Ω^* in the same figure, we conclude that the frequencies of the disk blade vibrations of the combined system in question must be approximately determined by the curves Ω_k' in Fig. 3. Consider, indeed, a point S_k of intersection of Ω^* with Ω_k. In such a point the frequency of the combined system is the same for wheel vibrations and for blade vibrations. If the system abandons both and choses the disk blade vibration instead, this happens, by virtue of the Rayleigh principle, at the

expense of the frequency; therefore the actual frequency at the blade length corresponding to S_k must be lower than that represented by the ordinate of S_k. So we arrive qualitatively at the curves Ω_k', $k = 2, 3, 4, \cdots$, in Fig. 3. [To obtain these curves *analytically*, it would be necessary, speaking roughly, to take only a certain part of the quantities $F_1|$ and $F_2|$ in the formula (48), see following section 8 of this paper.] The curves thus arrived at permit an estimation of the frequencies of the disk blade vibrations, which might be of sufficient accuracy for practical purposes (see section 9, semifinal paragraph).

8. Remarks Concerning More Accurate Frequency Computations of Disk Blade Vibrations.—The main purpose of the present section is to give an adequate idea of the amount of work, which is to be done in a more accurate computation of frequencies of disk blade vibrations.

We assume the vibration form of the wheel to be given by the formulæ

$$f(r) = 0 \quad \text{when} \quad r_0 \leqslant r \leqslant r_1; \qquad f(r) = (r - r_1)^2 r^s \quad \text{when} \quad r_1 \leqslant r \leqslant a, \tag{49}$$

while the vibration form of the blade shall be given by the formula (22), r_1 and s being the two parameters of the problem. By virtue of (49) we will have

$$f(r) = 0 \quad \text{when} \quad r = r_1; \quad df(r)/dr = 0 \quad \text{when} \quad r = r_1.$$

Introducing (49) into (34), (35) we will find the functions $F_i(\chi, s)$ of section 5 to be replaced by certain new functions $\varphi_i(\chi, s)$ respectively. The expressions for the energy amounts of the blade system are given by the formulæ (29), (30), but the constants occurring in the latter two formulæ are to be computed from a new set of equations replacing the equations (23)–(26). This new set of equations, in which an upper dash designates division by a^{s+2}, and a double upper dash—division by a, so that f.i. $\bar{A} = Aa^{-(s+2)}$, $\bar{r}_1 = r_1/a$, etc., is [for equ. (51) of this new set see section 6]

$$\bar{A} + \bar{B}a + \bar{C} = (1 - \bar{r}_1)^2, \tag{50}$$

$$\bar{B}a - \bar{C}pa - \bar{\beta}\bar{C}(pa)^2 = s + 2 - 2(s+1)\bar{r}_1 + s\bar{r}_1^2, \tag{51}$$

$$(1 - \nu^2)f\bar{C}(pa)^2 = s_2 - 2s_1\bar{r}_1 + s_0\bar{r}_1^2, \tag{52}$$

$$-(1 - \nu^2)f\bar{C}(pa)^3 = s_2' - 2s_1'\bar{r}_1 + s_0'\bar{r}_1^2, \tag{53}$$

wherein

$$\left.\begin{aligned} s_j &= \nu[(s+j)^2 - k^2] \\ &\quad + (1-\nu)(s+j)(s+j-1) \\ s_j' &= (s+j-2)[(s+j)^2 - k^2] \\ &\quad - (1-\nu)k^2(s+j-1) \end{aligned}\right\} (j = 0, 1, 2). \quad (54)$$

From the system of equations (50)–(54) the quantities $\bar{A}$, $\bar{B}a$, $\bar{C}$, pa are to be determined as functions of $\bar{r}_1$ and s (cf. Table 2 above, where the quantities just mentioned are computed as functions of s only). Substituting those functions of $\bar{r}_1$ and s into (29), (30) we obtain the energy amounts of the blade system as known functions of r_1 and s. The numerical computation of these functions is not difficult, but it is very tedious. Still more cumbersome is the computation of the function $\varphi_2(\chi, s, k)$, although the integrals involved are elementary and easily represented by finite expressions.

Having the energy amounts of the wheel and of the blade system, the frequency of the disk blade vibrations can be computed in the same manner as indicated in section 7, *A*, with the difference, however, that it is necessary now to determine the minimum of a function of *two* parameters, namely of r_1 and s. The calculations may be reduced to a certain extent by means of graphical integrations, but even then the amount of work required is a very large one.

In the actual conditions of steam turbine design it will be often possible to replace the rigorous method just outlined by the approximate solution, given in the preceding section.

9. Remarks Concerning Simplifications in the Analysis.— The above methods of frequency computation are based upon certain simplifications of the problem: not only is the disk treated in non-rotating conditions, but the effect of the twisting and tapering of the blades is not taken into consideration, and the influence of the shrouds is disregarded.

The following remarks are intended to show that no fundamental modifications will be necessary in order to bring the method given above into conformity with the requirements of the practical steam turbine design.

It is known that the centrifugal forces cause only slight changes in the frequencies obtained for disks at rest; these changes are easily calculated in practical problems, and it is, therefore, not necessary to introduce these forces here.

As to the twisted and tapered form of the long steam turbine blades, the assumption of a substantially undisturbed *axial* vibrating motion of the blades (in the axial disk blade vibration of the entire combined system), notwithstanding the twisted form of the blade, is supported, to a considerable degree at least, by some earlier experiments.[25] The axial motion of the blades is obviously enforced by the elastic resistance of the shrouds to other additional vibrations, as f.i. to torsional vibrations, of the blades.

The conclusion concerning the purely axial vibration of the blade groups represents an important and apparently well justified simplification of the problem. To appreciate its importance it is only necessary to compare the axial blade vibrations with the tangential blade vibrations, in which the shrouds undergo bending, and the bending of the shrouds influences in turn the boundary conditions of the blade at the outer end of the latter. Effects of this kind do not occur in the axial vibrations considered here.

It follows then that the formula (48) does not require any fundamental changes in the case of twisted and tapered blades with shrouds. More specifically, the formula (48) is true in the latter case as well, with the only changes in the expressions $[s]$ and $\{s\}$, which are to be computed now in some other way than before. Namely, the cross section area of the blade and the moment of inertia of this area are no more constants, but certain functions of the distance r from the turbine axis; this means that those expressions $[s]$ and $\{s\}$ will have to be determined from certain integrals, which require some more complicated computing work. (The pure blade vibrations, represented by the curve Ω^* in Fig. 3, can be obtained from these integrals in the limiting case $r_1 = a$, $F_1| = F_2| = 0$, see preceding section.) These integrations are, however, not as cumbersome as those occurring in the *disk* vibration problem as such. It was, therefore, considered as justified to eliminate here the additional problem of computation of the quantities $[s]$ and $\{s\}$ for twisted and tapered blades from (a more elaborate) discussion. As to the shrouds, their influence will

[25] See f.i. E. Soerensen, Berechnung von Eigenschwingungszahlen von Dampfturbinenschaufeln, in "Werft, Reederei, Hafen," 1928, pp. 73 and 76. The conception of the purely axial vibration of the blade groups in the problem considered finds no contradiction in the publications mentioned in the above footnotes either.

be represented by an additional mass element at the outer end of the blade.

The above considerations show that the analysis required for determination of the diagram Fig. 3 is essentially made up of such computing steps which are not unusual in the steam turbine practice; all of the operations involved are indeed familiar, in one or in another form, to the disk and blade designers. In computing the curves Ω_2, Ω_3, Ω_4, $\cdots$ in Fig. 3 it will be sufficient to determine, for each of these curves, two points only: the intersection point with the ordinate axis and the point corresponding to the blade length considered; by such two points each of the curves in question is practically well determined. To find the curve Ω^* it will be necessary to determine several points, but the computations are, as already mentioned, not as tedious as in the problem of wheel vibrations.

Thus in summarizing we may state that substituting the approximate diagram Ω_2, Ω_3, Ω_4, $\cdots$, Ω^* for the rigorous representation by the (analytically determined) curves Ω_2', Ω_3', Ω_4', $\cdots$ a considerable reduction of the problem is obtained, even in the case of twisted and tapered blades, without giving up the correct conception of the vibration actually occurring in the combined system in question. In many practical cases the approximate diagram (Ω_2, Ω_3, Ω_4, $\cdots$, Ω^*) will give the desired information to the question of resonance danger. If f.i. the impulse number for $k = 3$ nodal diameters is represented by the straight line $N_3 = 1000$, corresponding to $n = 3300$ r.p.m., roughly, we see at once in Fig. 3 that a blade length $\delta a = 0.43a$, with the other blade dimensions given above, does not produce any dangerous coincidence.

In the practical application the procedure of the computation would possibly undergo some further simplifications; it must be kept in mind, however, that the approximate solution must always conform to the principle of minimum with respect not only to the parameter s, but also to the additional parameter r_1, whatever form the simplified method may otherwise assume, if the approximate solution is to be correct at all.

10. Remarks Concerning the Disk with Heavy Rim.—In the case of a disk with heavy rim,[26] the discrepancies between

[26] See W. C. Heckman, l.c., p. 668.

theory and experiment are again due to an inadequate assumption concerning the characteristics of the vibration form of the disk. While in the original theory as represented by Kirchhoff the vibration form as such includes all kinds of boundary conditions, the vibration form as used in the Rayleigh Ritz method is always of a special character confined to a particular type of vibration; so the vibration form r^k introduced by Stodola and generally used in the frequency computations in the steam turbine practice, can be applied only to cases similar to that of a *free* boundary at $r = a$, which is the special case mostly treated in text and handbooks. The case of a disk with heavy rim, however, approaches more or less the case of a disk with an outer boundary *clamped in*. The vibration form of a disk with heavy rim will again be

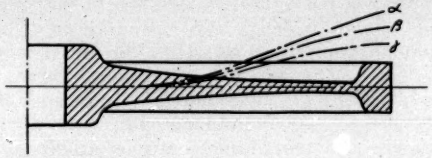


FIG. 8.

determined by the principle of the least ratio of deformation energy to kinetic energy in the entire system, giving the least possible frequency. That least ratio cannot be expected with a vibration form in which the heavy rim is forced to undergo deformation in such a high degree as required by the formula $r^k \sin k\vartheta$, where k designates the number of nodal diameters and ϑ is the azimuth angle. In reality the vibration form of such a disk with heavy rim will be of a character illustrated by the curves $\alpha, \beta, \gamma, \cdots$ in Fig. 8, approaching the case of a rim clamped in (see above). If a vibration form of this type will be substituted and properly varied in the frequency computations, which should be carried out by means of graphical integrations, it can be expected that the discrepancies between computation and experiment will disappear in this problem as well. This particular problem being, however, of less practical importance, we shall restrict ourselves to the above general remarks.

THE SPECIAL FORM OF CORIOLIS' COMPOUND CENTRIPETAL ACCELERATION DUE TO THE ROTATING EARTH.

BY

W. S. KIMBALL, Ph.D.,

Lt. Comdr., U. S. N. R., U. S. Naval Academy, Annapolis, Maryland.

INTRODUCTION.

A new derivation of Coriolis' normal acceleration due to the rotating earth by vector methods shows it to consist of two terms, $C = 2\omega_L v$, where $\omega_L = \omega \sin \phi$ is the component of the earth's rotation at latitude ϕ, and v is the velocity; and $K = 3\omega_L v_r$, where v_r is the velocity of the tangential acceleration field, i.e., a retarded field of energy per unit mass, $E_r = v_r^2/2 = v^2/2 - H$, being the difference between the energy of motion and the energy H of a harmonic field representing energy stored normal to the acceleration field in a circular path such as on the rotating earth maintains constant gradient speed with no tangential acceleration, and showing harmonic motion for its straight line projection. Since energy so stored balances a force field in a steady state, the circular path is referred to as a potential energy circle.

The classical formula for Coriolis' acceleration, $2\theta' v$ is checked by the above expressions: $2\theta' v = 2\omega_L v + 3\omega_L v_r$, where $\theta' = \omega_L + \frac{3}{2}\omega_L(v_r/v)$ is the complete time rate of change of direction and $\Delta\omega_L = \frac{3}{2}\omega_L(v_r/v)$ is that part of the rate of direction change that is due exclusively to the tangential and retarded acceleration field of velocity v_r, whose energy E_r is the remainder of the impressed field energy after deducting H, the energy that contributes nothing to the tangential acceleration, being stored on a potential energy circle and balanced by the effect of the earth's rotation.

The new expressions for Coriolis' force are applied to motions in a uniform parallel force field along the earth's

surface to evaluate the hitherto unsolved problem of finding the path equation of unit mass starting from rest as it accelerates and curves across the field until it reaches constant gradient velocity normal to the field. The work done against the field by the normal force $C = 2\omega_L v$ is found to be equal to the work done against it by the Coriolis term due to tangential acceleration $K = 3\omega_L v_r$. The path equation in this uniform force field is also derived in case of an initial velocity directly against the field and of magnitude equal to the field's gradient velocity.

1. THE POTENTIAL ENERGY CIRCLE.

The inertial circle is well known in meteorology as the path followed in the absence of pressure gradient where the Coriolis' force $C = 2\omega_L v$ balances the centrifugal force:

$$2\omega_L v = \frac{v^2}{r} : r = \frac{v}{2\omega_L}, \tag{1}$$

showing the radius of curvature r in terms of the velocity v and $\omega_L = \omega \sin \phi$ the angular velocity of the earth at latitude ϕ.

Furthermore it is well known that constant gradient velocity is maintained in a steady state in a circular path around symmetric high or low pressure areas, and hence represents energy stored that serves to balance these pressure areas. Thus, it can be referred to as potential energy, capable of doing work as soon as the balancing pressure is removed, and the circular path is a kind of potential energy circle. The equilibrium equation (2) for this anticyclonic (clockwise) circular path here considered is:

$$f = 2\omega_L v - \frac{v^2}{r}, \tag{2}$$

where f is the horizontal force per unit mass, represented by the pressure gradient normal to the circular orbit on which the energy is stored.

2. CORIOLIS' COMPOUND CENTRIPETAL ACCELERATION ON THE EARTH ROTATING AT CONSTANT ANGULAR VELOCITY $\omega_L = \omega \sin \phi$ HAS TWO PARTS: (1) $C = 2\omega_L v$, DUE TO THE CONSTANT ANGULAR VELOCITY AND A GIVEN SPEED, v, AND (2) $K = 3\omega_L v_r$, DUE TO TANGENTIAL ACCELERATION OF ENERGY $\frac{1}{2}v_r^2$ WHICH IS THE DIFFERENCE BETWEEN THE ENERGY OF MOTION AND THE ENERGY STORED NORMAL TO THE FORCE FIELD ON THE POTENTIAL ENERGY CIRCLE.

Consider a vector $s = ru_r$ in the tangent plane to the earth at o in latitude ϕ. The time derivative of s is:

$$\frac{ds}{dt} = \frac{dr}{dt} u_r + r \frac{du_r}{dt} = vu_r + r\theta' u_\theta, \tag{3}$$

where the time derivative of the unit vector u_r is given by dividing its differential shown in Fig. 1 by dt. The second derivative of $s = ru_r$ is:

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} u_r + 2v \frac{du_r}{dt} + r \frac{d^2u_r}{dt^2}$$

$$\doteq \left(\frac{dv}{dt} - r\theta'^2 \right) u_r + 2v\theta' u_\theta + r\theta'' u_\theta, \tag{4}$$

where the time derivative of the unit vector u_θ is given by dividing its differential shown in Fig. 1 by dt, and hence the derivatives of the unit vectors are:

$$\frac{du_r}{dt} = \theta' u_\theta; \qquad \frac{du_\theta}{dt} = -\theta' u_r; \qquad \frac{d^2u_r}{dt^2} = \theta'' u_\theta - \theta'^2 u_r, \tag{5}$$

where θ' and θ'' are first and second time derivatives of the angle θ.

At the origin o, where $r = 0$, the acceleration (4) reduces to:

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} u_r + 2v\theta' u_\theta, \tag{6}$$

where the normal force per unit mass $2v\theta' u_\theta$ will be recognized as Coriolis' compound centripetal acceleration.

We desire to specialize (6) and find the special form which Coriolis' force takes when due exclusively to the earth's rotation at *constant* angular velocity $\omega_L = \omega \sin \phi$. To do

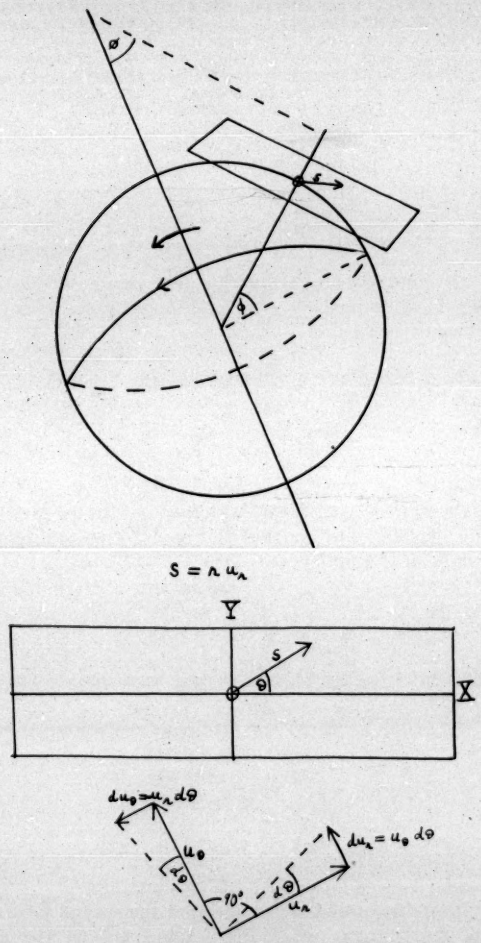


FIG. 1. Unit vectors and their increments in the tangent plane on the surface of the earth.

this replace θ' in (5) by this *constant* rate of change of direction:

$$\frac{du_r}{dt} = \omega_L u_\theta; \qquad \frac{du_\theta}{dt} = -\omega_L u_r. \tag{7}$$

Substitute in (4) and get:

$$\frac{d^2s}{dt^2} = \frac{dv}{dt}u_r + 2v\omega_L u_\theta - r\omega_L{}^2 u_r = \frac{dv}{dt}u_r + Cu_\theta, \tag{8}$$
$$C = 2v\omega_L,$$

where the term in r drops out at the origin where $r = 0$. Here the Coriolis' force is due entirely to motion in a field rotating at the constant earth's rate of rotation ω_L, and so is due entirely to the earth's rate of spin, being the familiar normal force involved in the circular gradient paths whose force equation is (2).

Where tangential acceleration occurs, as in (8), there is, however, an additional term besides $C = 2v\omega_L$ which must be included in the Coriolis force to give the expression (6) completely. Thus, it is clear that a negative constant tangential acceleration $-a$, yielding $v_r = -at$ in time t would involve a rate of change of direction due to the rotating earth and in opposition to (7). Thus (7) are correctly substituted for (5) to give the Coriolis acceleration (8) for constant speed on a circle according to (2), but an additional term is needed where tangential acceleration occurs, because this involves a time rate of change of direction $\Delta\omega_L$ in addition to (7). Thus instead of (7) and (8) we have:

$$\frac{du_r}{dt} = u_\theta(\omega_L + \Delta\omega_L); \qquad \frac{du_\theta}{dt} = -u_r(\omega_L + \Delta\omega_L); \tag{9}$$

$$\left.\begin{aligned} &\frac{d^2s}{dt^2} = \frac{dv}{dt}u_r + 2v(\omega_L + \Delta\omega_L)u_\theta = \frac{dv}{dt}u_r + Cu_\theta + Ku_\theta, \\ &\text{where} \quad K = 2v(\Delta\omega_L), \qquad \theta' = \omega_L + \Delta\omega_L, \\ &\qquad\quad C = 2v\omega_L; \end{aligned}\right\} \tag{10}$$

where θ' of (6) is replaced by the earth's constant rotation ω_L plus its increment $\Delta\omega_L$ due to tangential acceleration along

the path. Note that both C and K are included as parts of the Coriolis force (6).

We may compute K and $\Delta\omega_L$ of (10) by differentiating (8), using (7) again, and then integrating along a straight path: Thus:

$$\left.\begin{aligned}\frac{d^3s}{dt^3} &= \frac{d^2v}{dt^2}u_r + 3\frac{dv}{dt}\omega_L u_\theta - 3v\omega_L{}^2u_r - r\omega_L{}^3u_\theta \\ &= \frac{d^2v}{dt^2}u_r + 3\frac{dv}{dt}\omega_L u_\theta - 3v\omega_L{}^2u_r\end{aligned}\right\} r = 0 \quad (11)$$

is rate of change of acceleration where the changes in direction are due exclusively to the earth's constant rotation, ω_L according to (7).

Two and only two partial integrations of (11) are possible in case each is required to give the Coriolis normal acceleration terms, *exclusively* due to the earth's rotation. First, we may reverse the process of differentiation and integrate using (7) as before and obtain (10) which is the same as (8) with K added as a constant of integration, and $C = 2\omega_L v$ comes in as the Coriolis acceleration due to constant rates of direction change (7).

Or secondly we may integrate (11) along a straight path:

$$\frac{du_r}{dt} = \frac{du_\theta}{dt} = 0 \quad (12)$$

with zero direction changes, whereby the integration process itself introduces no additional normal acceleration, so that we have merely the sum of the rates of change (11) due exclusively to the earth's constant rotation ω_L:

$$\left.\begin{aligned}\frac{d^2s}{dt^2} &= \frac{dv}{dt}u_r + 3v_r\omega_L u_\theta - 3r\omega_L{}^2u_r + Cu_\theta, \\ \frac{d^2s}{dt^2} &= \frac{dv}{dt}u_r + 3v_r\omega_L u_\theta + Cu_\theta = \frac{dv}{dt}u_r + (C+K)u_\theta, \\ K &= 3v_r\omega_L,\end{aligned}\right\} \quad (13)$$

where the term in r is zero at the origin, and $C = 2\omega_L v$ is the constant of integration for the partial and rectilinear integration using (12).

Comparison of (13) with (6) and (10) gives the Coriolis force and the rate of change of direction due to the constant earth's rotation, including the extra term due to acceleration along the path:

$$C + K = 2v\theta' = 2v\omega_L + 3v_r\omega_L \begin{cases} \theta' = \omega_L + \frac{3}{2}\omega_L \dfrac{v_r}{v} \\ \Delta\omega_L = \frac{3}{2}\omega_L \dfrac{v_r}{v}. \end{cases} \tag{14}$$

Note that v_r is the actual velocity of the retarded acceleration field that survives the lopping off of the normal rates of direction change (7) by the substitution of (12), and is thus obtained by the lopping off of energy of the potential sort that is stored at constant speed against a balancing force field on a potential energy circle of equilibrium equation (2), and which hence contributes nothing to the actual acceleration field of velocity v_r. Furthermore this lopped off energy of constant direction change (7) and constant magnitude required by (2) is recruited from the total energy under *rectilinear* motion according to (12), and hence has the character of the straight line projection of motion in a circle at constant speed, i.e. it is harmonic motion of energy H determined by the boundary conditions of a particular problem:

$$v_r = \sqrt{v^2 - 2H}, \tag{15}$$

where the harmonic energy H deducted from the total energy is stored in equal [1] amounts as motion on the potential energy circle that balances the impressed field according to (2), and hence contributes nothing to the actual reduced acceleration that yields velocity (15).

We shall now use this new expression (14) for Coriolis' force to solve the hitherto unsolved problem of finding the path of unit mass starting from rest at the origin in a uniform parallel horizontal field on the eath's surface as its velocity increases up to rectilinear gradient velocity normal to this parallel force field. First, we note that this gradient line for this gradient velocity is at a characteristic distance from the origin depending on the uniform field strength.

[1] See eqs. (29) and (30) below.

3. A PARTICLE STARTING FROM REST IN A UNIFORM FIELD, a, OF ACCELERATION ON THE EARTH'S SURFACE, WILL ATTAIN GRADIENT VELOCITY, $v = \sqrt{2ab}$ ACROSS THE FIELD AT A CHARACTERISTIC DISTANCE, b, ALONG THE FIELD FROM THE ORIGIN; WHERE $b = a/8\omega^2 \sin^2 \phi$.

It is well known that where an atmospheric pressure gradient along the earth's surface is maintained in a steady state the wind velocity, v, at right angles to it, needed to balance it is given by Coriolis' force:

$$a = 2v\omega \sin \phi = 2v\omega_L, \tag{16}$$

where the acceleration a, is force per unit mass, and $\omega \sin \phi = \omega_L$ is the component of the earth's rotation at latitude ϕ and

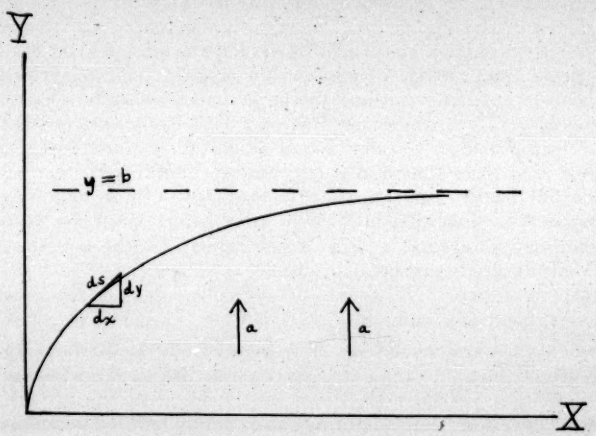


FIG. 2. The path of a particle starting from rest in a uniform field, a, and the gradient line, $y = b$, which it approaches asymptotically.

when the field of force in question is a uniform field with parallel lines of force as indicated in Fig. 2, then this so called gradient velocity of the wind will be along the straight line such as $y = b$, perpendicular to this field a.

The distance which the particle, starting from rest, will travel in direction of the field before attaining this gradient velocity may be easily computed, using the tangential force

along the path:

$$F_t = a \sin \alpha = a \frac{dy}{ds} = \frac{d}{ds} \frac{v^2}{2}. \tag{17}$$

Here α is evidently the inclination of the path shown in Fig. 2 with its sine expressible in terms of the shown differentials. If we integrate (17) along the path, we have:

$$\frac{v^2}{2} = \int a \sin \alpha ds = \int a dy = ay; \qquad v^2 = 2ay. \tag{18}$$

When (18) refers to the final steady gradient velocity, its y will be called b, and the equation (18) will then refer to the same state of affairs represented by (16), so that we may eliminate v between the two equations (16) and (18):

$$\begin{aligned} v_b^2 &= 2ab, \\ a &= 2\omega_L v_b = 2\omega_L \sqrt{2ab}, \\ b &= \frac{a}{8{\omega_L}^2} = a/8\omega^2 \sin^2 \phi, \end{aligned} \tag{19}$$

which shows the distance b, along the uniform field, a, which a particle (of air) starting from rest must travel in order to attain gradient velocity $v_b = \sqrt{2ab}$ according to (18).

Note that the distances y and b along the field which determine by (18) a given velocity are independent of the path from the origin, because $\sin \alpha$ is inversly proportional to ds, for a given dy, so that distance along the path is immaterial to velocities (18) determined by y, and these in this respect are like velocities down a frictionless inclined plane due to gravity.

4. THE ENERGY AND FORCE EQUATIONS OF A PARTICLE STARTING FROM REST IN A UNIFORM FIELD ON THE EARTH'S SURFACE.

The components of energy per unit mass in the x and y directions at (x, y) on the path where α is the angle of inclination are:

$$\begin{aligned} \frac{{v_x}^2}{2} &= v^2 \frac{\cos^2 \alpha}{2} = \int_0^x (2\omega_L v + K) \sin \alpha dx, \\ \frac{{v_y}^2}{2} &= ay - \frac{{v_x}^2}{2} = \frac{v^2 \sin^2 \alpha}{2} = \int_0^y [a - (2\omega_L v + K) \cos \alpha] dy. \end{aligned} \tag{20}$$

Here we have introduced the new normal acceleration K, in addition to the acceleration $C = 2v\omega \sin \phi = 2v\omega_L$, and due as we have seen to acceleration along the path. Differentiation of (20) with respect to x and y gives:

$$\begin{aligned} v\frac{dv}{dx}\cos^2\alpha + v^2\cos\alpha\frac{d}{dx}\cos\alpha &= (2\omega_L v + K)\sin\alpha, \\ v\frac{dv}{dy}\sin^2\alpha + v^2\sin\alpha\frac{d}{dy}\sin\alpha &= a - (2\omega_L v + K)\cos\alpha. \end{aligned} \tag{21}$$

From the geometry we have:

$$\sin\alpha = \frac{dy}{ds} = \frac{y'}{\sqrt{1+y'^2}}; \qquad \cos\alpha = \frac{dx}{ds} = \frac{1}{\sqrt{1+y'^2}},$$

and hence

$$\frac{d}{dx}\cos\alpha = \frac{-y'y''}{(1+y'^2)^{\frac{1}{2}}} = \frac{\tan\alpha}{r} = -y'^2\frac{d}{dy}\sin\alpha, \tag{22}$$

and

$$\frac{d}{dy}\cos\alpha = \frac{-y''}{(1+y'^2)^{\frac{1}{2}}} = \frac{1}{r},$$

where r, the radius of curvature is taken as positive since y'' is seen to be negative. We may replace the derivatives of the sine and cosine in (21), by the radius of curvature given by (22) and transposing, obtain the force equations:

$$\begin{aligned} F_x &= \cos^2\alpha\frac{d}{dx}\frac{v^2}{2} = \left(2\omega_L v + K - \frac{v^2}{r}\right)\sin\alpha, \\ F_y &= \sin^2\alpha\frac{d}{dy}\frac{v^2}{2} = a + \left(\frac{v^2}{r} - 2\omega_L v - K\right)\cos\alpha. \end{aligned} \tag{23}$$

Multiply the first of (23) by $\cos\alpha = dx/ds$, and the second by $\sin\alpha = dy/ds$ and add to get the tangential force equation (17):

$$F_t = F_x\cos\alpha + F_y\sin\alpha = (\cos^2\alpha + \sin^2\alpha)\frac{d}{ds}\frac{v^2}{2}. \tag{24}$$

Carry out the differentiations of (23), using $v^2 = 2ay$, and divide the first by $\sin\alpha$ and the second by $\cos\alpha$ to get from

each equation the normal force equation:

$$a \cos \alpha = 2\omega_L v + K - \frac{v^2}{r}, \tag{25}$$

or

$$d = 2\omega_L v + K = a \cos \alpha + \frac{v^2}{r}.$$

The second of (25) is shown in Humphrey's "Physics of the Air," Chapter VII, Fig. 39.

5. THE NORMAL FORCE K DOES THE SAME AMOUNT OF WORK AGAINST THE UNIFORM FIELD, a, AS $2\omega_L v$ DOES.

Two interesting conclusions can be drawn from equations (20) to (25). The first is that no such path as that of Fig. 2 leading asymptotically to $y = b$, is possible in the absence of K. For one thing the bracket integrand of the second of (20) would always be positive (if K was zero because of (19)), so that v_y would not be zero at $y = b$. Or again (25) shows that the path could not cross above $y = b$ and then return to $y = b$ after a point of inflection above b because if v^2/r were zero or negative (25) could not be satisfied above b since by (19) $2\omega_L v > a \cos \alpha$ in that region.

Also, (25) shows that K is of the same order of magnitude as $C = 2\omega_L v$ and does the same amount of work against the field "a" as the path goes from 0 to $y = b$. To show this, replace v^2 by $2ay$ as in (18) and $1/r$ by $d/dy \cos \alpha$ as in (22) and replace [2] $C + K$ by $2C$, i.e. $C = K$.

$$2ay \frac{d}{dy} \cos \alpha + a \cos \alpha = 2C = 4\omega_L \sqrt{2ay}. \tag{26}$$

This is a linear differential equation in $\cos \alpha$ with integrating factor $1/\sqrt{y}$ which integrates to:

$$2a\sqrt{y} \cos \alpha = 4\omega_L y \sqrt{2a} \tag{27}$$

or

$$a \cos \alpha = 2\omega_L \sqrt{2ay} = 2\omega_L v,$$

[2] Equation (26) is not correct since with zero curvature on $y = b$ it does not satisfy (19), but the argument using (26) shows that C and K do equal work against the field "a."

which satisfies (19) at $y = b$ and $\cos \alpha = 1$ along the gradient line, showing that twice the amount of work done by C is required to cause the 90° deflection between $y = 0$ and $y = b$, since $2C$ of (26) replaces the deflection force of (25).

6. THE RETARDED ACCELERATION FIELD FOR A UNIFORM IMPRESSED FIELD ON THE ROTATING EARTH IS ALWAYS LINEAR.

The energy for the general retarded acceleration field given by (15) becomes for the special case of a uniform parallel force field whose energy is (18):

$$E_r = \frac{v_r^2}{2} = \frac{v^2}{2} - H = ay - \frac{ay^2}{b}. \qquad (28)$$

Where the harmonic second degree term H is uniquely determined by the boundary requirements which are evidently that v_r shall be zero at $y = 0$ and $y = b$, the former where the acceleration starts to do work, and the latter where a steady state with constant v is reached, and the tangential acceleration has finished its work.

Note also that $H = ay^2/b$ represents the projection on the x axis or x component E_x of energy of motion $v_b^2/2 = ab$ at constant speed on the potential energy circle of radius b shown in Fig. 3.

$$H = E_x = \frac{v_b^2}{2} \cos^2 \alpha = ab \frac{(y^2)}{b^2}, \qquad (29)$$

being the integral of the centripetal force component parallel to the x axis.

Since this v_b^2/b is a normal force, it must do an equal amount of work of opposite sign (resultant zero) in the y direction:

$$-\int \frac{v_b^2}{b} \cos \alpha dy = -2a \int \frac{y}{b} dy = -\frac{ay^2}{b} = -H. \qquad (30)$$

The work done (30) by the centripetal force in the minus y direction against the field "a" and deposited according to (29) as motion at right angles to the field on the potential energy circle evidently represents a total loss of tangential acceleration energy thus now reduced from ay by this H ac-

cording to (28). No matter what supply source (30) is taken from (such as ay of (28)) its deposit in equal amounts (29) at right angles to the field "a" is mechanically equivalent to the deposit of a component (29) of motion at constant speed $v_b^2 = 2ab$ on the potential energy circle and as such measures the loss to the acceleration field (28), because such motion

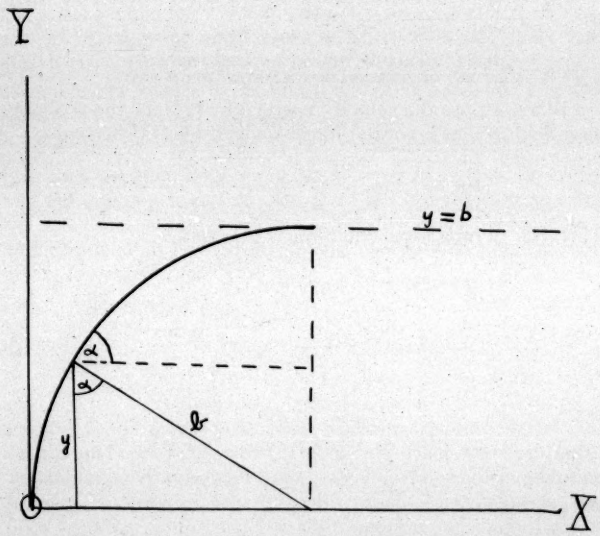


FIG. 3. The potential energy circle of radius b, on which energy, taken from the uniform field, a, is stored as motion at right angles to the field.

is mechanically balanced according to (2) when it occurs on the rotating earth.

Note also that the force causing the retarded field energy (28) is given by the derivative of (28):

$$F = \frac{d}{dy} E_r = a - \frac{2ay}{b} = a - \frac{v^2}{b}. \qquad (31)$$

Furthermore since the impressed field is uniform and of

constant "a," the combination of "a" with the linear $d/dy(H) = v^2/b$, is also always a linear harmonic field and its potential energy for this case is:

$$P = -\int Fdy = \frac{ab}{4} - ay + \frac{ay^2}{b} = \frac{a}{b}\left(y - \frac{b}{2}\right)^2, \quad (32)$$

where the constant of integration $ab/4$ is the maximum value of (28) at $y = b/2$.

7. THE EQUATION OF THE PATH ACROSS THE RETARDED FIELD OF ACCELERATION, STARTING FROM REST.

When we use the velocity v_r given by (28) for the acceleration field, the new normal force K given by (13) becomes:

$$K = 3\omega_L v_r = 3\omega_L \sqrt{2ay\frac{(b-y)}{b}}.$$

Then the normal force equation (25) becomes:

$$\begin{aligned} d &= 2\omega_L v + 3\omega_L v_r = a\cos\alpha + \frac{v^2}{r}, \\ &= 2\omega_L\sqrt{2ay} + 3\omega_L\sqrt{2ay - \frac{2ay^2}{b}} \\ &= a\cos\alpha + 2ay\frac{d}{dy}\cos\alpha, \end{aligned} \quad (33)$$

using (22) to replace $1/r$, and (18) to replace v. This differential equation is linear in $\cos\alpha$ and its derivative, and has an integrating factor $1/\sqrt{2ay}$:

$$\sqrt{2ay}\frac{d}{dy}\cos\alpha + \frac{a\cos\alpha}{\sqrt{2ay}} = 2\omega_L + \frac{3\omega_L}{\sqrt{b}}\sqrt{b-y},$$

$$\sqrt{2ay}\cos\alpha = 2\omega_L(y+b) - \frac{2\omega_L}{\sqrt{b}}(b-y)^{\frac{1}{2}}.$$

Here the constant of integration, $2\omega_L b$ is required by the boundary condition (19). Since $a = 8\omega_L^2 b$, we have:

$$\cos\alpha = \tfrac{1}{2}\left[\frac{y+b}{\sqrt{by}} - \frac{(b-y)^{\frac{1}{2}}}{b\sqrt{y}}\right] = \frac{1}{\sqrt{1+y'^2}}. \quad (34)$$

Note that both (33) and (34) check the boundary con-

dition (19) taking $y = b$ and $\cos\alpha = 1$, and the curvature $d/dy \cos\alpha = 0$. To obtain the path equation in terms of x and y, we need to integrate (34) again, but this is inconvenient and unnecessary since the coordinates α and y are sufficient for accurate plotting.

Equation (34) shows how rapidly the path swings across the field to the right under operation of the normal forces C and K. Thus for $y = b/4$, $\cos\alpha = (10 - 3\sqrt{3})/8 = .6$ and $\alpha = 53° 8'$, showing more than one third of the 90° deflection in a quarter of the distance along the field. At $y = (b/2)$, $\cos\alpha = (3\sqrt{2} - 1)/4 = .81$ and $\alpha = 35° 50'$, and for $y = \frac{3}{4}b$, $\cos\alpha = (13/8\sqrt{3}) = .938$ and $\alpha = 20° 16'$.

Just as $v = \sqrt{2ay}$ given by (18) is independent of the path (34) and depends only on y, so $v_r = \sqrt{2ay - (2ay^2/b)}$ given by (28) is independent of the path (34) and angles α and for the same reason, and it is like v, a velocity along the path (34) and obtained by integrating along the path the component of the force given by (31):

$$\frac{v_r^2}{2} = \int \left(a - \frac{2ay}{b} \right) \sin\alpha ds$$

$$= \int \left(a - \frac{2ay}{b} \right) dy = ay - \frac{ay^2}{b} = E_r. \quad (35)$$

8. THE EQUATION OF THE PATH WHERE THE INITIAL VELOCITY IS $V_b = \sqrt{2ab}$ DIRECTLY AGAINST THE UNIFORM FIELD, a.

The equation of the path (34) for the particle starting from rest is unique, whereas with an initial velocity v_0, the path equation will depend on the direction and magnitude of v_0 being usually given in terms of elliptic functions. These difficult cases are outside the scope of the present article, but as a further illustration of $K = 3\omega_L v_r$, we shall treat the case referred to by the present paragraph heading which happens to be easily solved.

In case of an initial velocity v_0, equation (18) above has to be replaced by

$$\frac{v^2}{2} = \int a \sin\alpha ds = \int a dy = ay + \frac{v_0^2}{2}, \quad (36)$$

$$v^2 = 2ay + v_0^2 = 2ay + 2ac,$$

where c is a length defined in terms of $v_0 = \sqrt{2ac}$. Since the force equation (19) for gradient velocities still holds as a boundary condition, we have from (36):

$$\left.\begin{aligned} a &= 2\omega_L v_b = 2\omega_L\sqrt{2ab} = 2\omega_L\sqrt{2a(y_0 + c)}, \\ y_0 &= b - c = b - \frac{v_0^2}{2a}. \end{aligned}\right\} \tag{37}$$

Equation (37) shows that no matter what the direction of v_0, its corresponding gradient velocity $v_b = \sqrt{2ab}$ ultimately reached in the steady state, will always be along the line $y_0 = b - (v_0^2/2a)$ as indicated by Fig. 4 and reached after some such path that approaches $y_0 = b - c$, assymptotically, as shown above $y = 0$.

For the special case where $c = b$ and $v_0^2 = 2ab = v_b^2$, the gradient velocity for the uniform field, a, equals the initial velocity and hence, the gradient line $y_0 = b - c$, given by (37) becomes $y_0 = b - b = 0$, which is the x axis. This will be approached ultimately, asymptotically, by all paths having $v_0 = \sqrt{2ab}$ for their initial velocity at the origin, no matter what their initial direction. If $v_0 = \sqrt{2ab}$ is directed along the positive x axis, its path will be the straight line $y = 0$. This is the simplest possible case, representing always complete equilibrium between gradient force and uniform field according to (37). We now consider the special case where this $v_0 = \sqrt{2ab}$ is directed along the negative y axis, whose path reaches down to $y = -b$ in the 3d quadrant and then swings back up to the line $y = 0$, which it approaches asymptotically as indicated in the lower part of Fig. 4. For this case the linear field of tangential acceleration is included between the lines $y = 0$ and $y = -b$ instead of $y = 0$ and $y = +b$ as with (28), and the potential energy is given by:

$$\left.\begin{aligned} P &= \frac{a}{b}\left(y + \frac{b}{2}\right)^2 = \frac{a}{b}\left[(b + y)^2 - b(y + b) + \frac{b^2}{4}\right] \\ -\frac{\partial P}{\partial y} &= a - \frac{2a(y + b)}{b} = a - \frac{v^2}{b} = -a - \frac{2ay}{b} \end{aligned}\right\}. \tag{38}$$

The energy constant $E = ab/4$ is the same as with (32) for

the linear field extending over the same interval b, and the velocity is given by (36) taking $c = b : v^2 = 2a(y + b)$. The kinetic energy of this acceleration field is given by subtracting (38) from the energy constant $E = ab/4$, or by

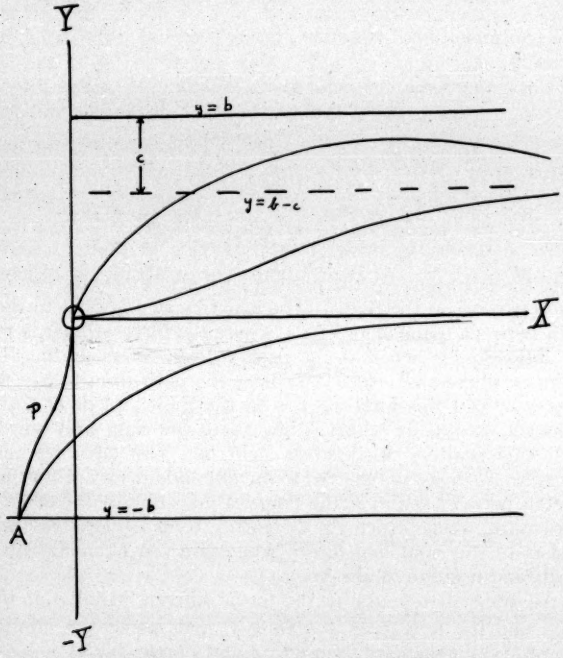


FIG. 4. The paths where there is initial velocity at the origin in a uniform field, a, and the gradient lines which they approach.

integrating the force equations given by (38):

$$\left.\begin{aligned} E_r = -ay - \frac{ay^2}{b} = \frac{v_r^2}{2} = a(b + y) - \frac{a}{b}(b + y)^2 \\ v_r^2 = \frac{-2ay}{b}(b + y) \end{aligned}\right\}. \quad (39)$$

This evidently is real only between $y = 0$ and $-b$ according to Fig. 4. And hence the normal force due to acceleration is:

$$K = 3\omega_L v_r = 3\omega_L \sqrt{\frac{-2ay}{b}(y+b)}. \tag{40}$$

The normal force equation (25), corresponding to (33) becomes:

$$d = \frac{v^2}{r} + a\cos\alpha = \pm 2\omega_L v + 3\omega_L v_r,$$

$$2a(y+b)\frac{d}{dy}\cos\alpha + a\cos\alpha \tag{41}$$

$$= \pm 2\omega_L\sqrt{2a(y+b)} + \frac{3\omega_L}{\sqrt{b}}\sqrt{-2ay(y+b)}.$$

Note the two signs for the positive and negative velocities in the first term on the right. The negative sign applies to the path between 0 and A in Fig. 4, where the force $-2\omega_L v$ is to the left and up, whereas the positive sign refers to the remainder where the velocity is along the path from $y = -b$ to $y = 0$, and this force $2\omega_L v$ is to the right and down and supports $K = 3\omega_L v_r$ which is always to the right and down with this positive acceleration field, a. The left member terms are each always exerted to the left and up, except for the stretch between 0 and P the point of inflection of the downward path, along which the curvature term is negative and pushes to the right and down, whereas $a\cos\alpha$ remains upwards and positive as always.

An integration factor for this linear differential equation in $\cos\alpha$ and its derivative is $1/\sqrt{b+y}$, and, using the minus sign of (41) for the path from 0 to A and simplifying by means of $a = 8\omega_L^2 b$, (41) becomes:

$$\left.\begin{aligned} 2b\sqrt{b+y}\frac{d}{dy}\cos\alpha + \frac{b\cos\alpha}{\sqrt{b+y}} &= -\sqrt{b} + \tfrac{3}{2}\sqrt{-y} \\ \cos\alpha = \frac{y\sqrt{-y} - y\sqrt{b}}{2b\sqrt{b+y}} &= \frac{1}{\sqrt{1+y'^2}} \end{aligned}\right\}, \tag{42}$$

where the last expression comes from integrating and solving

for $\cos\alpha$. This is the equation of the path from o to A, which like (34) we shall not attempt to express in terms of x and y by integrating again. Note that $\cos\alpha$ is always positive since y is negative and between o and $-b$, and has a zero value at $y = -b$ as well as $y = 0$, indicating a path such as shown in Fig. 4 from o to A, with positive slope and parallel to oY at its two ends. The maximum value of $\cos\alpha$ is at the point of inflection where the curvature $d/dy \cos\alpha = 0$. Making this substitution in the first of (42) gives:

$$\frac{b\cos\alpha}{\sqrt{b+y}} = -\sqrt{b} + \tfrac{3}{2}\sqrt{-y}, \tag{43}$$

when $\cos\alpha$ is eliminated between (42) and (43) we have the critical condition giving y for the point of inflection, which may be put in the form, using $z^2 = -y/b$:

$$2z^3 - z^2 - 3z + 2 = (z-1)(2z^2 + z - 2) = 0, \tag{44}$$

$$z = \frac{\sqrt{17}-1}{4}; \qquad z^2 = \frac{(\sqrt{17}-1)^2}{16} = \frac{9-\sqrt{17}}{8} = \frac{49}{81} = \frac{-y}{b},$$

$$z = \frac{7}{9}, \qquad y = \frac{-49}{81}b.$$

If these critical values of y and z are used in (43) we have for the maximum $\cos\alpha$ at the point of inflection P:

$$\cos\alpha = (\tfrac{3}{2}z - 1)\sqrt{1-z^2} = \frac{\sqrt{32}}{54} = .105, \tag{45}$$

If we take half of this maximum $\cos\alpha$, as a rough average between $y = 0$ and $y = -b$ where it is zero in both cases, we have for x at A, using $\bar{\alpha} = 87°$:

$$x = -b\cot\alpha = -0.052b, \tag{46}$$

which shows that in going against the field of acceleration from o to A, the wind particle is only deflected to the left by about 1 part in 20.

For the return path from $y = -b$ at A to the line $y = 0$ approached asymptotically, we have to integrate (41) using the plus sign for the force $2\omega_L v$ which now supports rather

than opposes the acceleration term. Using $a = 8\omega_L{}^2 b$, as before, we now have instead of (42):

$$2b\sqrt{b+y}\,\frac{d}{dy}\cos\alpha + \frac{b\cos\alpha}{\sqrt{b+y}} = \sqrt{b} + \tfrac{3}{2}\sqrt{-y}. \qquad (47)$$

This integrates to:

$$\left.\begin{aligned} 2b\sqrt{b+y}\cos\alpha &= \sqrt{b}(y+2b) - (-y)^{\frac{3}{2}} \\ \cos\alpha &= \tfrac{1}{2}\left(\frac{(y+2b)\sqrt{b} - (-y)^{\frac{3}{2}}}{b\sqrt{b+y}}\right) \\ &= \tfrac{1}{2}\left(\frac{(y+2b)\sqrt{b} + y\sqrt{-y}}{b\sqrt{b+y}}\right) \end{aligned}\right\}. \qquad (48)$$

Here the constant of integration $2b\sqrt{b}$ satisfies the boundary conditions $\cos\alpha = 0$ at $y = -b$, and $\cos\alpha = 1$ at $y = 0$. It will be noted that using $b + y = z$, and $b - z = -y$ to substitute for y in (48) gives an expression in z which is identical with that given by (34) in terms of y, thus showing that (48) and (34) are identical paths. This checks the correctness of the theory since both paths represent the same situation, i.e. zero initial velocity under operation of the uniform field a, the only difference being that with (34) we start from the origin, whereas with (48) the "initial velocity" is zero at A of Fig. 4.

PHOTOLYSIS OF THE *d*-GLYCOSIDES: α BENZYLFRUCTOFURANOSIDE, β BENZYLFRUCTOPYRANOSIDE, AND α AND β PHENYL, BENZYL AND β PHENYLETHYL GLUCOSIDES; AND THE BEARING OF THE DATA UPON THE TRANSFER OF ENERGY WITHIN MOLECULES.*

BY

LAWRENCE J. HEIDT.

Many natural organic coloring matters and substances with physiological activity are found in plants as glycosides together with their aglycones. Sunlight and ageing have long been known to change their color and dying properties.[1] The experiments reported here [2] show that light also liberates the aglycone from the reducing sugar. One can smell the phenol liberated when phenylglucosides absorb λ254 mμ, and at the same time material is produced which reduces the alkaline cupric reagent of Shaffer, Hartmann and Somogyi (SHS). Changes in optical activity also occur. The quantum yields were measured and together with the other data cast light also on the identication of α and β isomers and the transfer and interconversion of energy within molecules.

Materials: The glucose, fructose and glycosides were the remains of samples kindly put at my disposal by Professor C. B. Purves of the Organic Department after the bulk of them had been used by us in a systematic study of the thermal

* Contribution from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, No. 494.

[1] (*a*) Bancroft, Philosophy of Permanent Colours (1813); (*b*) Perkins and Everest, The Natural Organic Colouring Matters (1918), Longmans Green and Co., New York; (*c*) Armstrong and Armstrong, The Glycosides (1931), Longmans Green and Co., New York.

[2] A preliminary report was given by the author in the *J. Am. Chem. Soc.*, 61, 2981 (1939). In the table presented there, the light flux per min. should have been given as 6.6×10^{15} instead of 8.6×10^{15} and Hypo, ml. for β-benzylfructopyranoside was $8.2 \times 11/2 = 45.0$ instead of $6.4 \times 11/2 = 35$. Also on p. 2981 in line 11, "acid which" should read "and."

rates of hydrolysis of the glycosides.[3] They were white crystalline specimens whose melting points and optical activities (the latter are given in the table) agreed with the accepted values and whose colorless solutions were neutral to bromthymol blue. They were stored in the dark in a vacuum over anhydrone.

The fructose was kindly purified by Dr. F. B. Cramer. His procedure is given below in some detail because it removed traces of impurities which absorbed light, thereby decreasing the depth of the λ240 mμ minimum of the absorption spectrum of fructose.

The fructose was obtained as C.P. Special Levulose sp. rot., − 92°; ash 0.05 per cent.; moisture 0.1 per cent. Its aqueous 10 per cent. solution was treated with special decolorizing charcoal,[4] allowed to stand and subsequently filtered. The filtrate was evaporated in a vacuum at not over 30° to 80 to 90 per cent. solids. The solids were dissolved in absolute ethanol, seeded and recrystallized at 0°. These crystals were washed with absolute ethanol and dried at 55° in a vacuum over P_2O_5. The whole procedure was repeated thrice even though no detectable change occurred in the m.p., sp.rot. or appearance of the crystals.

The materials used in the analytical procedures were of Analytical Reagent or C.P. quality.

The actinometer was made up of dry recrystallized specimens of the best materials obtainable. It was a water solution containing in one liter, 0.0017 and 0.0040 moles of uranyl oxalate and oxalic acid respectively.

Apparatus[5] *and experimental procedures*[2,6] have been described for the most part. Photolyses were made with monochromatic light of λ254 mμ. This wave-length falls near the center of the first broad absorption maxima of the

[3] (*a*) Heidt and Purves, J. Am. Chem. Soc., **60**, 1206 (1938) and (*b*) a forthcoming publication by us.

[4] The charcoal had been prepared by digestion on a steam bath with 12% HCl and subsequently washed with distilled water and filtration by gravity and suction until the loose powder was acid free. This removed material which appeared as a white solid upon evaporation of the acidic extract.

[5] Heidt, *Science*, **90**, 473 (1939).

[6] (*a*) Heidt, *J. Am. Chem. Soc.*, **61**, 3223 (1939); (*b*) Heidt, *J. Chem. Phys.*, **10**, 297 (1942).

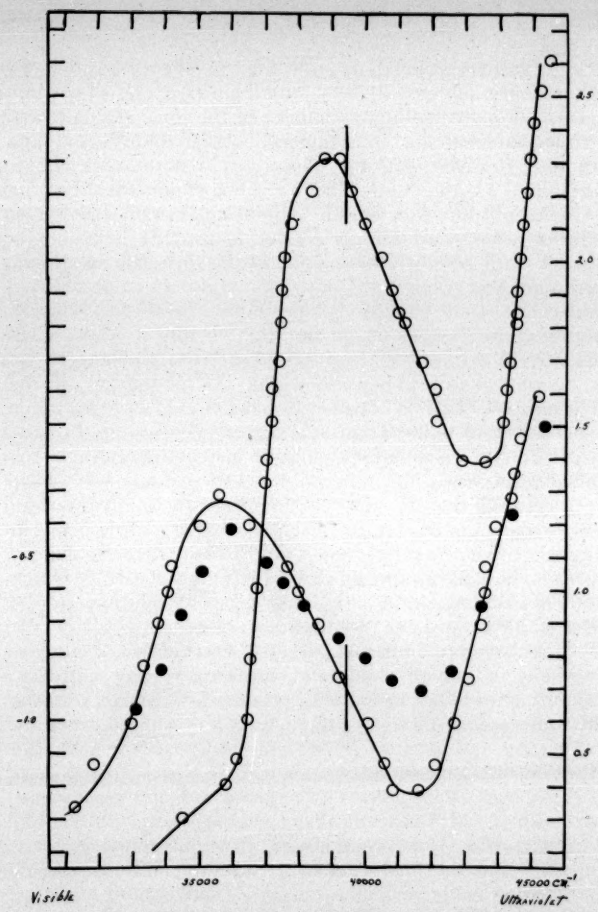


FIG. 1. The vertical lines in the plot are at λ254 mμ, the wave-length of the light with which the solutions were photolyzed.
Left Ordinate: $\text{Log}_{10}\,\epsilon$ for Fructose (lower curve).
Right Ordinate: $\text{Log}_{10}\,\epsilon$ for benzylfructosides (upper curve).
$\epsilon=[\log_{10}(I_0/I)]/cl$ where c is moles of solute per liter of solution and l is the depth, in centimeters, of the light absorbing layer.

aryl-glycosides studied as indicated in Fig. 1 so that its extinction is affected little by small shifts in the absorption spectra brought about by changes in the ionic strength and temperature of the environment. Also, the glucose and fructose produced during photolyses absorb a negligible fraction of $\lambda 254$ mμ under the prevailing conditions.

The light flux was measured by means of the actinometer solution whose composition is given above. Its volume was always made equal to the volume of the glycoside solution it preceded and replaced. Both solutions were photolyzed under the same conditions and both absorbed all but a negligible fraction of the actinic light before it could reach the stirrer.

A decrease of pH accompanied the photolyses of the glycoside solutions as noted in Table 1 of ref. 2. This made it necessary to buffer them in order to prevent the decrease in pH from proceeding far enough to increase measurably the dark hydrolysis.

Increasing the pH, however, increased the intensity of an amber color produced during the photolyses. A pH between three and four, therefore, was found to suit best the experiments. This also happens to be the range of pH at which Mathews and Jackson [7] observed fructose solutions to be most stable toward heat (optimum stability at pH 3.3).

Molar acetate buffer at pH 3.5 was chosen because it interferes in only a small and measurable way with the estimations of reducing material produced, because it absorbs only a negligible fraction of the light of $\lambda 254$ mμ absorbed by the system and because it has practically no effect on the quantum yields measured.

Analytical Procedures: The reducing material was measured with the SHS reagent as outlined in part elsewhere.[3,6,8] Dark and photolyzed portions of the stock solutions were analyzed simultaneously under the same conditions. Each analysis was made with an accurately fixed amount of about 5 ml of the SHS reagent. This was thoroughly mixed with the neutralized sample to be analyzed whereupon the mixture was placed in boiling water at constant level for fifteen min-

[7] Mathews and Jackson, *Bur. Standards J. Res.*, 11, 619 (1933).
[8] Somogyi, *J. Biol. Chem.*, 117, 771 (1937).

utes and then transferred quickly to running tap water, also at constant level, for five minutes after which it was titrated at once.

The 0.002 molar hypo used in the titrations was fresh and slightly alkaline. One ml of it was equivalent in the unbuffered solutions to 5.9×10^{-5} g. fructose or 5.5×10^{-5} g. glucose on the average when one fourth the SHS reagent had been reduced. In solutions buffered with molar acetate these values were 6.1×10^{-5} and 5.7×10^{-5} respectively.

Percentage errors in the estimations were largest when little and almost all of the reagent was reduced. The first was avoided by introducing a measured amount of the reducing sugar or partially hydrolyzed glycoside; the second by diluting the sample to be analyzed.

Changes in optical activities were measured to 0.02° for yellow sodium light through four decimeters of solution at 21.5°. The polarimeter tube was fitted with a small bore and held about 10 ml. of solution. Dark and photolyzed portions were read as near the same time as possible and were staggered when necessary. Thereafter, the solutions were kept on ice until the SHS estimations could be made.

Analysis of the actinometer solution was made by electrometric titration as formerly.[6a] The permanganate solution used in these titrations and indicated by *B* in Table 1 contained 0.00183 formula weight of $KMnO_4$ per kilogram of solution.

Absorption Spectra: These were obtained at 25° by the null photographic method and are portrayed in Fig. 1. The extinction coefficient, ϵ, is defined in Fig. 1. The lower curve is for fructose which had been purified by recrystallization and drying less than a day before. The solution contained 1.9093 g. fructose in 4.0 g. water and was 2.039 molar as calculated from the density of levulose solutions given by the equations of Jackson and Mathews.[9] Circles represent the data of the author, and dots the data obtained by others [10] and approached in this research by solutions made from crystals which had stood for a year in dry air out of direct

[9] *Bur. Standards J. Research*, **8**, 403 (1932).

[10] Kwiecinski and Marchlewski, *Bull. Int. Acad. Polon. Sciences Lettres*, Series A, 330 (1929). Their solution was half molar.

sunlight. The m.p. and sp.rot. of the crystals during this time had not changed measurably and their water solution was colorless; hence, only a minute amount of material can be responsible for the large increase in absorption about λ240 mμ.

The upper curve in the Fig. is for β benzylfructopyranoside and α benzylfructofuranoside (0.270 g. in 50 ml.). One solution of the β compound contained 0.573 g. and the other 0.1401 g. in 25 ml. Beer's law was obeyed and the molar acetate buffer at pH 3.5 had no measurable effect upon ε down to 250 mμ.

The upper curve in Fig. 1 also represents, within the limits of error, the data obtained for benzyl glucosides [11] and benzyl alcohol.[11,12] This coincidence suggests, as already noted,[11] that the aglycone is the absorbing center of the glycoside in the region of the spectrum covered in Fig. 1 for the remaining parts of these glycosides though present in sucrose are nearly as transparent as water down to 200 mμ.[6]

ORIENTING EXPERIMENTS WITH SOLUTIONS OF β BENZYLFRUCTOPYRANOSIDE.

β benzylfructopyranoside was chosen for the orienting work because conditions which minimize its subsequent and side reactions insure that these reactions occur to a lesser extent with the more stable glucosides and glucose. Its decomposition was followed, in this preliminary work, mainly with the SHS reagent. Comparisons were made at the same fraction of the fructoside decomposed which was usually less than 10 per cent.

The experiments showed that the photochemical efficiency for the photolysis of this glycoside varies less than ± 5 per cent. when (1) the concentration of buffer is changed from 0 to 0.1 to 1.0 molar acetate and from 0 to 0.1 *M* phosphate at pH values of 3.5, 4.5, 5.5 and 6.5 and temperatures from 8 to 25°; (2) the light intensity is varied by a factor of over one hundred [2]; (3) the concentration of fructoside is varied between 0.14 and 0.9 g. in 50 ml. The main reaction, therefore, is unimolecular and its freedom from any meas-

[11] Ramart-Lucas and Rabate, *Bull. Soc. Chem. France*, **5**, 2, 1596 (1935).

[12] Marchlewski and Wyrobek, *Bull. Int. Acad. Polon. Sciences Lettres*, Series A, 109 (1929).

urable dependence upon temperature indicates that enough energy of the absorbed photon, roughly 110 kcal. per einstein, gets into the reactive center to produce the measured reaction without requiring any measurable thermal activation.

The amber color which develops during photolysis was found to be caused by oxidation by air of one or more of the products. This was found by photolyzing the glycoside solutions under their own pressure and excluding air as explained in ref. 6*b*. No amber color appeared then until air was admitted. Fructose solutions develop a similar discoloration in air but do so much more slowly; hence if the amber color is caused by the fructose produced, the latter must be more labile than normally which seems plausible in view of the large amount of energy available for the photochemical reaction compared to that required thermally, roughly 26 kcal., to set the fructose free.

The products of the main photochemical reaction appear to be the same as those formed by thermal hydrolysis in acid, namely, fructose and benzyl alcohol. This is suggested by the following observations. First, the fraction of the glycoside hydrolyzed is the same whether calculated from the SHS estimations or from the changes in optical activities based on fructose as the measured product as shown by the data in ref. 2 and in Table 1 presented here. Second, no muta-rotation of the photolyzed samples is observable under the prevailing conditions as expected if fructose is the sugar produced. Third, the odor and frothing of photolyzed and hydrolyzed solutions are the same during the course of the SHS estimations. In fact, the only apparent difference between hydrolyzed and photolyzed solutions is the amber color which develops in the latter and is intensified during the SHS estimations.

The reactive center indicated by these experiments is the hemiacetal oxygen bridge linking the aglycone which forms the benzyl alcohol with the rest of the glycoside which forms the fructose. Very little of the main reaction, however, is caused by photons which this center directly absorbs. This was tested directly by illumination with $\lambda 254$ mμ of 0.02 molar β *methyl*-fructopyranoside which contains the non-absorbing methyl in place of the benzyl group and is almost

as easily hydrolyzed thermally [3] but which produced photochemically less than 3 per cent. of the reducing material formed from the benzyl compound under the same conditions. It can be concluded, therefore, that most of the energy producing the photochemical reaction is transferred to the reactive center from the aglycone.

QUANTUM YIELDS.

Quantum yields, ϕ, for the photochemical breakdown of the fructosides and glucosides are summarized in Table 1. These are based on the ΔSHS values and all the light of λ254 mμ absorbed by the system. In these experiments the water solutions contained in 1 liter, 0.02 formula weight of glycoside and one mole of acetate buffer which held the pH at 3.5 ± 0.1. Twelve ml. samples were irradiated in quartz at 10 ± 2°. They were vigorously stirred and absorbed all but a negligible fraction of the actinic light before it could reach the stirrer.

Two stars in the table identify the glycosides which differ from the others in that the optical activities of their solutions change sign during hydrolysis. They also are the least optically active and exhibit other properties not shown by the others which are pointed out nearer the end of the text.

α_i gives the specific rotation for the samples not irradiated. They were calculated, as indicated, from the observed rotations divided by eight times the grams of glycoside dissolved in 50 ml.

$M\alpha_i$ gives the molecular rotations. The formula weights, M, for phenyl, benzyl and phenylethyl glucosides and fructosides were taken as 256, 270 and 284 respectively.

α_f gives the specific rotation for 1 gram of the glycoside completely hydrolyzed in the same solution. This was calculated from the equation $\alpha_f = 180\alpha^*/M$ where α^* is the observed specific rotation of glucose (52°) or fructose (− 91°) in the same environment.

$\Delta\alpha$ gives the fractions of glycosides photolyzed as calculated from the observed changes in α. These fractions were calculated under α_f from $\alpha_i - \alpha_f$, and under 0 from $\alpha_i - 0$ as the change expected for complete reaction. A star in these columns indicates that in these representative cases no

TABLE I.

Summary of Data Giving Gross Quantum Yields, ϕ, for the Breakdown of Aryl-glycosides by Light of λ254 mμ.

Other data also are included for comparison. Values of ϕ at the same fractions of the glycoside decomposed were reproduced within ±3%.

	$\alpha_i/100$	$M\alpha_i$	α_f	Fraction Photolyzed Calculated from Δα using			Minutes Photolyzed	Δg of *B* per Min.	ϕ	Thermal Half Life in Min. at 75° in 0.1*N* HCl
				α_f	ϕ	ΔSHS				
d fructosides										
β benzyl pyranoside	−2.84/2.16 = −1.32	−46,400	−61	0.11	0.18	0.10	184	1.0	0.019	0.63
				.33	.54	.34	960	.83	.014	
**α benzyl furanoside	0.97/2.16 = 0.45	12,200	−61	.07*	.16	.16	970	.29	.018	.063
				.18	.42	.43	3870	.29	.012	
d glucopyranoside										
α phenyl	3.74/2.05 = 1.82	46,600	37	.11	.09	.10	1030	.39	.008	250
				.26	.20	.24	2560	.67	.004	
**β phenyl	−1.46/2.05 = − .715	−18,300	37	.09	.14	.12	1700	.25	.008	730
α benzyl	2.93/2.19 = 1.34	36,200	35	.10	.07	.13	630	.37	.018	7000
				.31	.22	.35	2670	.37	.011	
**β benzyl	−1.21/2.17 = − .56	−15,100	35	.05	.08	.12	620	.35	.018	3500
				.12	.21	.20	1500	.33	.013	
α-β phenylethyl	2.88/2.26 = 1.27	36,200	33	.09	.07	.11	1760	.14	.015	>7000
				.22	.16	.22	5720	.11	.011	
**β-β phenylethyl	−0.71/2.29 = − .31	− 8,800	33	.04	.08	.10	1000	.20	.015	
				.18*	.38	.35	4280	.33	.008	

change took place in $\Delta\alpha$ in 3 days in the dark at 21°. Thus one made sure that in every case the mutarotation of the liberated reducing material was completed before the optical activity was measured.

ΔSHS gives the fractions of glycosides photolyzed as obtained from the SHS estimations on the dark and photolyzed samples. The product reducing this reagent was taken to be the sugar formed on hydrolysis.

Δg of B/min. gives grams of permanganate solution, 0.00183 molar, equivalent to the oxalate photolyzed per min. in the actinometer.

ϕ = (0.02) (12) (0.6) (fraction photolyzed)/(0.00183) (2.5) (min. photo) (Δg of B/min.) = 31.4 (fraction photolyzed)/(min. photo) (Δg of B/min.). 0.6 is the value taken for the quantum yield for the actinometer. The factor 2.5 in the denominator equates formula weights of $KMnO_4$ to oxalate photolyzed per minute in the actinometer.

The half lives given in the last column of the table are rough values for the thermal hydrolyses in tenth normal hydrochloric acid at 75°. These were calculated from data some of which has not yet been published.[3b]

The photochemical efficiencies for light absorbed only by the glycosides are possibly as much as two or three times ϕ. They are in doubt by this much because ϕ decreased much more than expected from the inner filter affect of the liberated aglycone. It was, however, possible to reproduce ϕ within ± 3 per cent. so that *relative* values of ϕ at the same fraction of glycoside photolyzed possibly represent *relative* values of the net photochemical efficiencies with this accuracy.

DISCUSSION.

The identification of α and β isomers is based on the premise that the α isomer has the more positive specific rotation. The α fructoside, however, behaves photochemically more like β than α glucosides for like the latter its photochemical products are apparently optically inactive. The relative half lives of the α and β isomers in the two series also are inverted. The bearing of these facts on the identification of these isomers will be dealt with in detail later[3b] when the results of the quantum yield measurements, mo-

lecular refractions, and constants for the rate equations for the thermal hydrolysis can be compared and Hudson's Rules for molecular refraction considered. The molecular refractions are put in Table I to avoid repetition in the forthcoming publication [3b] of much of the other data in this table.

The photochemical efficiencies for the photolyses measure the product of two probabilities. One of these is the probability of sufficient of the absorbed energy getting to the reactive center to cause reaction. The other is the probability of reaction occurring when the reactive center possesses this energy. The latter is the so-called transition probability and has been found to be very near unity for the thermal decomposition of hydrogen iodide. The data above show that it can be considered constant for reactions of the same type; otherwise the quantum yields in Table I would not have been the same for benzyl glucosides and fructosides at the same fraction photolyzed in view of the fact that the fructoside hydrolyzes thermally many thousand times more rapidly than the corresponding glucoside. This equality of the transition probabilities implies that the large differences in the thermal rates of hydrolyses of these glycosides are caused primarily by differences in their energies of activation as has been found to be the case.[3b]

Quantum yields based on the SHS estimations are seen to be the same for (1) the α and β isomers, (2) pyranosides and furanosides, and (3) glucosides and fructosides. In fact ϕ at the same fraction photolyzed apparently depends only on the structure of the aglycone and along with the absorption spectra supports the hypothesis, that *the main photochemical reaction* measured in these experiments *results from an intramolecular transfer of absorbed energy from the aglycone to the hemiacetal oxygen bridge with practically no part played by the remainder of the glycoside.* This, in spite of the dominating role played by the latter in influencing the thermal hydrolyses in acid as can be seen from the differences in the half lives recorded in Table I.

Quantum yields based on changes in optical activities and calculated from either one of the two sets of $\Delta\alpha$ values are seen by simple proportion to differ as much for the α and β isomers as for fructosides and glucosides of the same aglycone.

If ϕ is sensitive to such changes, one would expect it to depend like the half lives upon structural differences outside the aglycone; hence relative changes in ϕ and the half lives should roughly parallel each other. This, however, is not the case for the changes in ϕ are small in all cases while the half lives differ by less than a factor of three for α and β isomers, and by more than a factor of ten thousand for fructosides and glucosides of the same aglycone.

Comparing the two sets of $\Delta\alpha$ values with the ΔSHS values suggests that optically active photochemical products are formed from either the unstarred or starred glycosides but not from both. This must be accepted with certain reservations, however, because of the findings of Purves and Hudson.[13] They observed, in some thermal reactions of glycosides not in water, changes in optical activity which did not always correspond to the reducing material produced. They even observed anomalous increases in optical activity without production of reducing material. No such effects were observed in aqueous solution although a search was made for them in this research.

Conversion of energy of the absorbed photon into potential and kinetic energy of the units of the molecules occurs, of course, otherwise there could have been no photolysis. The absorption spectra and quantum yield measurements both suggest that this interconversion of energy takes place in the aglycone.

The intramolecular transfer of energy from the aglycone to the reactive center requires that quantum yields change with changes in structure between the absorbing and reactive centers. Such changes do exist, as can be seen in Table I, where an increase in the number of like bonds traversed decreased ϕ in passing from benzyl to β phenylethyl glycosides. However, the phenyl glycosides, contrary wise, have the lowest instead of the highest values for ϕ, so that other factors such as resonance, polar character and the directing influence of the attached group also play an important part in the reaction when the light absorbing phenyl group is attached directly to the reactive center.

[13] *J. Am. Chem. Soc.*, **59**, 49 and 1170 (1937).

SUMMARY.

Direct absorption of light by glycosides dissolved in water has been discovered to produce changes in optical activities and formation of material which reduces the Shaffer, Hartmann, Somogyi reagent. Comparison of these measurements indicates in many cases the same products as are formed by hydrolysis in acid. The β glucosides and α fructoside, however, apparently give optically inactive photolytic products.

Absorption spectra were obtained for α benzylfructofuranoside and β benzylfructopyranoside, and were found to be the same in the near ultraviolet as those previously obtained for benzyl glucosides and benzyl alcohol.

The absorption spectrum for freshly purified fructose gives a deeper minimum about $\lambda 240$ mμ than the previously reported values. The latter, however, were approached when the dry crystals had stood in air for several months although no change occurred in their sp.rot., m.p. or general appearance.

All the glycosides studied were white crystalline compounds. Less than five per cent. of the ultraviolet light of $\lambda 254$ mμ they absorb produces reaction; nevertheless, the apparatus and procedures available were adequate to study the reaction conveniently in monochromatic light of this wave-length.

The absorption spectra, the products of photolysis, and the quantum yields, the latter based on the reducing material formed, lead to the hypothesis that the photochemical reaction is produced by an intramolecular transfer of absorbed energy from the aglycone to the hemiacetal oxygen bridge which is the reactive center. The efficiency of this transfer is, as expected, greater for benzyl than for phenylethyl glucosides but phenyl glucosides, contrariwise, are the least efficient.

The structure and energy possessed by the glycoside outside the aglycone have no measurable effect on the efficiency of the photochemical reaction when followed by measuring the reducing material produced although they change up to one hundred thousand fold the rates of the thermal hydrolyses in acid.

Sodium Light For Microscopic Flaw Detection.—Sodium light, which has proved so effective in the prevention of night traffic accidents, is being used in several important war plants for surface inspection by microscope, because of its efficiency in the detection of pits, cracks, and flaws in materials. The sodium lamp is essentially an arc lamp, and differs from the incandescent in that it requires a special socket and individual control for each lamp. The unusual perception of detail under sodium illumination is due to the monochromatic nature of the light, according to H. A. BREEDING, of the General Electric Illuminating Laboratory. The eye, in common with other lenses, actually focuses only one narrow wave or color band at a time. Other colors in the beam tend to fog the picture. Thus, minute details may be lost in examining an object under a light which contains all wave bands, in contrast to an examination of the same object under monochromatic light which contains only one wave-length. "Probably because people through the centuries have looked at more green and yellow-green objects than those of any other color," Mr. Breeding explains, "the eye has gradually developed an affinity for light in this color band. Sodium light is golden yellow and very near the region of yellow-green in the spectrum where maximum eye sensitivity occurs. A given amount of energy from sodium light produces several times as much illumination as the same amount of energy in white light." As in the case of sodium lighting for night safety driving, monochromatic light for microscopic inspection is most effective when spread uniformly over a large area rather than concentrated in small bright patches. In this way an undistorted view of both size and shape of flaws present is obtained. For most effective results, the level of illumination should be comparatively high, especially when the contrast between flaw and surrounding material is low. It is helpful in searching for cracks and blow holes to treat the part with either a light-absorbing or light-reflecting dye, the dye depending on the nature of the surface to be inspected. The dyed surface should be cleaned thoroughly before inspection. This procedure increases the contrast between the flaw and object to the point where many times smaller flaws can be detected. In all cases it has been found necessary to block out as much sunlight as possible, in order not to destroy the monochromatic nature of the sodium illumination.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

POLARIMETRY, SACCHARIMETRY AND THE SUGARS.

The increasing applications of polarized light to the arts and sciences, especially in the field of carbohydrate chemistry, has led to many requests to the Bureau for information. In Circular C440 by Frederick J. Bates and Associates, an attempt is made to answer as far as possible such inquiries and to furnish information of a broader character and in greater detail than could be given by letter.

Emphasis has been placed on the applications of polarimetry in carbohydrate chemistry and indeed many pages are devoted to what might be considered the pure chemistry of the sugars. As a justification for the special applications of the polariscope given in this circular, it may be stated that carbohydrate chemistry could hardly have developed to the magnitude it has attained in recent years without the aid and guidance of the polariscope, and that many inquiries concerning the analysis, preparation and properties of the sugars have been directed to this Bureau.

The subject matter presented is divided into five parts. The first of these, consisting of 115 pages, covers an original mathematical treatment of the physical phenomena and a description and discussion of the physical equipment, such as polarimeters, saccharimeters, and accessory apparatus utilized in the study and applications of polarized light. The development of polarimetric equipment is discussed from the historical standpoint, with emphasis upon recent improvements. The text is illustrated by many diagrams and photographs which are accompanied by simple explanations of the physical concepts and deductions used.

Part II, consisting of 275 pages, relates to the analysis and properties of sugar products such as raw and refined sugars, honey and sirups. It includes an extensive discussion of the polarimetric and chemical methods utilized for the determina-

* Communicated by the Director.

tion of sucrose, invert sugar, reducing sugars, pentosans, dextrose, levulose, and other sugars. Sections are devoted to the important physical properties of the sugars and their solutions, notably, density, refractive index, color, turbidity, viscosity, acidity, electrical conductivity, solubility and boiling point.

Part III, consisting of 140 pages, is devoted to the preparation and properties of the sugars and their derivatives. It includes discussions of the optical activity of organic compounds and of the structures, configurations and reactions of the sugars. Methods are given in detail for the preparation of 24 important sugars and for numerous sugar derivatives. The test is supplemented by an extensive table which lists the important sugar derivatives and gives physical constants and literature references.

Part IV, consisting of 9 pages, gives general information concerning the standard samples of sucrose and dextrose issued by this Bureau and concerning tests of equipment which the Bureau is prepared to conduct.

Part V, consisting of 235 pages, contains 78 reference tables of numerical data and a resume of the work of the International Commission for Uniform Methods of Sugar Analysis and the United States Customs regulations governing the weighing, taring, sampling, classification and polarization of imported sugars and molasses.

The treatise of 800 pages contains in all 150 numerical tables, which provide convenient sources of reference data for workers in both technological and academic fields. The data given in certain of these tables has not been heretofore available and the numerous literature references should provide a rich source of information for all concerned with the development of polarimetry and sugar chemistry.

Copies of Circular C440, which is bound in blue buckram, are obtainable at $2.00 each from the Superintendent of Documents, Government Printing Office, Washington, D. C.

TESTING AND ACCURATE USE OF ABBÉ-TYPE REFRACTOMETERS.

The Abbé type of refractometer, with heatable double prisms and a compensator to permit the use of a white-light

source, is the most generally used instrument for commercial refractometry, especially for liquid media. For accurate work it is useful to consider refractometers as comparison instruments with which media of unknown refractive indices are referred to standard samples. Tests for various mechanical and optical defects are advisable, and in particular, the compensator should be examined for possible deviation of the rays of sodium light.

Either temperature control, or careful observation and measurement of existing temperature, is very important in refractometry of liquids because their indices decrease, on the average, about 4×10^{-4} per 1° C. increase in temperature. Usually, changes of less than $\pm 5°$ C. in the temperature of the instrument do not seriously affect its performance, but when refractometers are used at high prism-jacket temperatures it is necessary to know, and correct for, the temperature errors of the instrument. An alternate procedure, and a preferable one if a compensator is involved, is the use of a series of standard refractive-index samples for which the indices are known at the temperatures in question.

When attempting high accuracy on solid samples, the contact liquids should be so selected as to minimize their "wedge effect" errors and to insure good visibility of the interference fringes that are used for adjusting the wedge. In using liquid samples between the Abbé double prisms one must make certain that the critically refracted rays are not shielded because of excessive separation of the surfaces of the prisms and that the true border is not masked by systems of interference fringes. These become more troublesome as the ground glass surface of the illuminating prism is partially polished by repeated cleanings.

Dispersion measurements, as customarily made by means of the compensator on an Abbé refractometer, are necessarily much less reliable than the attainable precision. It is recommended that line-spectra sources be employed whenever accurate dispersions are desired, and it is preferable to dispense altogether with a compensator and calibrate the scale for each wave length that is to be used. These matters are discussed in detail by Leroy W. Tilton in the July number of the *Journal of the Optical Society of America*.

PHOTOELECTRIC MEASUREMENT OF COLOR.

Interest in the measurement and quantitative specification of color has grown rapidly in recent years because of the increased use of numerical designations of color in purchase specifications for paint, paper, ceramic, and other materials, and because of the increased attention given to the appearance of many finished articles sold on the market. The photoelectric tristimulus method of measuring color is simple and direct and its possibilities have not been examined until recently. Although new filter-photocell methods for measuring color have been announced frequently during the last 15 years, measurements have shown that the filters and photocells in most of these instruments were not properly chosen, as regards spectral character, for making the purported measurements of color.

In Circular C429, which has just been released, Richard S. Hunter describes a number of types of color measurements which can be made with any device having the source, three filters, and photocell which he has selected. Because the resultant three combinations of source, filter, and photocell are nearly equivalent spectrally to the ICI standard observer for colorimetry, measurements with them give valuable quantitative color information. However, as it was not possible to obtain combinations exactly equivalent to the ICI observer, these measurements are not in exact agreement with colorimetric values computed from spectrophotometric data. The suggested photoelectric tristimulus combinations are nevertheless well suited for measuring the color differences between spectrally similar specimens.

In order to measure color differences as small as those visible to trained inspectors of paint, textile, plastic, paper, and ceramic products, an instrument must have high precision. If the needed precision is available, a photoelectric tristimulus colorimeter may be used to measure: (1) ICI colorimetric values, x, y and Y, relative to those of a spectrally similar, calibrated standard, (2) values of α and β, components of the chromaticity departure from neutral, (3) amounts of color difference between pairs of spectrally similar samples, (4) amounts of color change accompanying fading, (5) whiteness

of white and near-white surfaces, and (6) the yellowness of white and near-white surfaces.

In giving examples of the measurement of some of these different properties and in describing the errors of color measurement to which the tristimulus method is subject, reference is made to operations with the author's recently developed multipurpose photoelectric reflectometer.

Copies of C429 are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 10 cents each.

HEAT OF COMBUSTION OF BENZOIC ACID.

Benzoic acid has been used for many years as a standard substance for calibrating bomb calorimeters, and has been selected as the primary standard for this purpose by the Standing Committee for Thermochemistry of the International Union of Chemistry. Such a use of benzoic acid requires an accurate value for its heat of combustion under the conditions of the bomb process.

Accurate measurements of the heat of combustion of benzoic acid have been reported by a number of observers in the last 35 years, but since impurities might affect the heat of combustion of different lots of benzoic acid by significantly different amounts, it has been the practice at the Bureau to determine the heat of combustion of each lot of standard-sample material.

A paper by R. S. Jessup (RP1499) in the October *Journal of Research* gives results of measurements of the heats of combustion of two lots of benzoic acid designated as standard samples 39*e* and 39*f*, and of two samples of specially purified benzoic acid which contained less than 0.01 mole per cent. of impurity.

The value obtained for the heat of combustion of benzoic acid at 25° C. is 26428.4 ± 2.6 international joules per gram mass when the sample is burned in a bomb under certain specified conditions. No significant difference was found between the heats of combustion of the specially purified samples and the standard-sample material. With one exception, results reported previously are in satisfactory agreement with those of the present work.

SELF-IGNITION TEMPERATURE OF CELLULOSE AND MAGNESIUM.

One of the fire-hazard properties of a material is defined as the lowest initial temperature from which, under favoring conditions, it will heat spontaneously until glow or flame occurs. Determinations thereof have been made at the Bureau of Standards for a number of solids and liquids. The self-ignition temperature of cellulose is of interest considering that it is the main constituent of a wide range of combustible materials. This point for nearly pure cellulose such as filter paper was found to be 232° C. (450° F.), ignition occurring after 7 to 9 hrs. of heating. With less time lag higher initial sample temperatures were required, for 3 hrs., 246° C.; 2 hrs., 253° C.; and for 1/2 hr., 280° C. Tests continued for 45 hrs. at 230° C. and 70 hrs. at 228° C. did not result in ignition. The lowest self-ignition temperature of cellulose such as absorbent cotton was determined as 266° C. (511° F.), the time lag being 2½ hrs. No ignition obtained with heating at 263° C. for 8 hrs. and 264° C. for 17 hrs.

The weight of the carbonized residue from tests in which ignition did not occur at temperatures a little below the lowest self-ignition temperature was from 25 to 40 per cent. of the original weight of the sample, the latter being 3.3 g. for filter paper and 1.3 g. for absorbent cotton, with volume of both of about 10 ml. This high weight of residue is further evidence of non-ignition, the weight of the ash after ignition being less than 1/2 per cent. of the original weight. Ignition was preceded by rapid self-heating with temperature rise up to about 600° C. when glow appeared. Air was passed through the sample of a rate deemed most favorable for self-heating.

The ignition temperature of magnesium is also of considerable interest on account of its extensive use in industry and for military purposes. For the pure material in various degrees of fineness the following gives general results from a large number of tests:

Fineness	Self-Ignition Temperature	
	° C.	° F.
Small pieces of magnesium ribbon	540	1004
Passing No. 30, retained on No. 50 screen	510	950
Passing No. 50, retained on No. 70 screen	508	946
Passing No. 70, retained on No. 200 screen	504	939
Passing No. 200, retained on No. 325 screen	492	918
Passing No. 325 screen	473	883

Determinations with samples containing 1 per cent. aluminum and about 1 per cent. combined of other impurities gave similar values within 10 to 15° C.

STRUCTURAL PROPERTIES OF "PHC" PREFABRICATED WOOD-FRAME CONSTRUCTION:

Building Materials and Structures Report BMS90 gives the results of tests made in collaboration with the Forest Service, U. S. Department of Agriculture, on 33 specimens representing "PHC" prefabricated wood-frame constructions submitted by the PHC Housing Corporation. These specimens included two wall, one floor and two roof constructions.

The wall specimens were subjected to compressive, transverse, concentrated, impact, and racking loads; the floor specimens to transverse, concentrated, and impact loads; and the roof specimens to transverse and concentrated loads. The transverse, concentrated, and impact loads were applied to both faces of the wall specimens. For each of the loads three like specimens were tested. The loads simulated those to which the elements of a house are subjected in actual service. For each increment of load, the deformation under load and the permanent set after its removal were measured. The results are presented in graphs and in tabular form.

Copies of BMS90 are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C. The price is 15 cents.

Butter Could be Made in Tropics From Butteroil and Dry Skim Milk.—Lack of refrigeration on ships or in storage warehouses need not keep the dairy industry from supplying "butter" to the armed forces and civilian population in tropical countries, say dairy scientists of the United States Department of Agriculture. Pure butteroil and skim milk powder—two dehydrated dairy products that contain all the food nutrients found in butter—could be shipped to tropical countries, held for a year if desired, and then made into good butter with the aid of a little cold water, according to the Bureau of Dairy Industry. Butter usually contains 80 per cent. fat (butteroil), about 1 per cent. skim milk constituents. The rest is water and salt. DR. GEORGE E. HOLM, who is in charge of the Bureau's fat-spoilage research, recently perfected a method of preparing the pure butteroil and packaging it so that it will keep almost indefinitely under extreme conditions. Skim milk powder is already produced and packaged by the industry. The Bureau has successfully packaged butteroil in 2 pound tin cans for demonstration purposes. Metal oil drums would be ideal containers for large scale shipments, but because tin cans and metal drums are both made of critical materials, the Bureau is now testing the possible use of wooden kegs. Dr. Holm long ago established the fact that fats do not spoil unless a certain amount of oxygen is present to combine with the fat. Recently he found that the usual procedure in packaging butteroil did not exhaust all the oxygen from the container. Using the proper physical conditions of agitation, temperature, and pressure to remove all the oxygen and other gases, and then sealing the oil in gas-tight containers will prolong keeping quality, he found. Butteroil properly prepared is a delicately flavored product that can be used in many of the ways that butter is used. It cannot properly be called dehydrated butter because dehydration implies that only the water is removed from the butter, whereas the curd as well as the water is removed from butter in making butteroil.

R. H. O.

THE FRANKLIN INSTITUTE.

STATED MEETING OF THE INSTITUTE, WEDNESDAY, OCTOBER 21, 1942.

The stated monthly meeting of The Franklin Institute, the first after the summer months, was held in the Hall of the Institute at 8:15 p.m., Wednesday, October 21st.

After the playing of the Star Spangled Banner, when the audience stood at attention, the meeting was called to order by the President, Mr. Charles S. Redding.

The Chairman stated that the minutes of the last meeting (held in May) had been published in full in the June issue of the JOURNAL and asked if any change should be made. There were no comments and the minutes were approved as printed.

The Chair then called upon Dr. Henry Butler Allen, Secretary of the Institute.

Dr. Allen reported with much satisfaction that since the last meeting of the Institute in May, eight hundred and twenty-five new members had been added to the rolls of the Institute, making the interesting total of just over five thousand members. He stated that the facilities of the Institute were especially valuable to people in service and pointed out that the technical library was of great use.

Dr. Allen also reported that from the comparatively small personnel of The Franklin Institute, six members had been inducted into the service, and that a Service Flag in Franklin Hall took cognizance of the fact.

He then asked members to send in information regarding their own or other members' activities in the armed service, in order that a record might be established in the Institute.

Mr. Redding then announced the speaker of the evening, Dr. Detlev W. Bronk, Professor of Biophysics, Director of the Johnson Research Foundation, Director of the Institute of Neurology, University of Pennsylvania.

Dr. Bronk spoke on "The Human Machine in Aerial War" and stressed the close relationship between the engineer and the physiologist who are working together to protect the human body from the strains imposed upon it by the aircraft developed for ever increasing altitudes and accelerations. [Dr. Bronk's paper will be published in full in the JOURNAL OF THE FRANKLIN INSTITUTE.]

The meeting was adjourned with a rising vote of thanks to the lecturer for a highly informative talk.

October 22, 1942.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, October 14, 1942.)

HALL OF THE COMMITTEE,
PHILADELPHIA, OCTOBER 14, 1942.

MR. JOHN B. KLUMPP *in the Chair.*

The following report was presented for final action:

No. 3101: Hydrodynamic Brake System for Hoists and the Like.

This report recommended the award of Edward Longstreth Medals, one each, to Robert Griffin De La Mater and William Schwemlein, of Parkersburg, West Virginia, "In consideration of the design and successful application of a fluid brake which has made practical the drilling of wells to depths hitherto possible only with great difficulty, and the accomplishment of this purpose with greater safety to life and property."

JOHN FRAZER,
Secretary to Committee.

MINUTE ADOPTED BY THE COMMITTEE ON SCIENCE AND THE ARTS IN REFERENCE TO THE DEATH OF MR. CLARENCE ARTHUR HALL.

THE COMMITTEE ON SCIENCE AND THE ARTS records in this Minute the loss by death on July 20, 1942 of one of its most loved and valued members, CLARENCE A. HALL.

He graduated from the University of Pennsylvania in 1896, with degree of Bachelor of Science in Chemistry. Immediately after graduation he became chemist for the Lake Superior Corporation at Sault Ste. Marie, Ontario, Canada. Only a short time prior, Wilson had discovered calcium carbide and had exhibited it at the Institute. His work at the Soo was largely in connection with the commercial production of calcium carbide. He was a pioneering chemist in that field.

He spent only a year or two at the Soo and was then transferred to Niagara Falls plant which is now the Union Carbide Company. He assisted there in the development of a continuous calcium carbide producing furnace.

His next connection was with the Pennsylvania Salt Company of Philadelphia. Then a short term with the Link-Belt Company of Philadelphia; viz., from May 16, 1910 until November 11, 1911.

From the Link-Belt Company he went to the Electric Storage Battery Company and there remained until his death.

He was elected to membership in the Institute in 1902, became a member of the Science and Arts Committee in 1906, serving as Chairman in 1926, and received in 1940 the Hoadley Certificate after 35 years of devoted and valuable service. He was appointed a member of the Library Committee in 1914 and served several years as its Chairman. He was elected a member of the Board of Managers in 1922 and continued on the Board until his death. He served on the Committee on Publications and was Chairman of the Bartol Research Foundation Committee. In 1939 he was elected Assistant Treasurer.

The Chairman of your Sub-Committee was a lifelong friend of Mr. Hall, having gone through the Central Manual Training High School and the University

of Pennsylvania with him. We were close pals in and out of school. In his school days, as in after life, he had that something we call personality, which made one feel glad to be with him. It was an honor to be his friend and a great pleasure to feel the warmth of his friendship. He was a splendid pal.

He was possessed of a keen and inventive mind—always in search for a new way or a new something. Thirteen United States patents were granted to him. Seven foreign patents and three United States patent applications are pending.

His service on the Committee on Science and the Arts was most loyal, devoted and helpful. Probably his outstanding traits were his willingness to be helpful and his absolute dependability when given an assignment.

The members of the Committee on Science and the Arts, wishing to record their sorrow in the loss of their beloved fellow member, instruct that this Minute be recorded in the proceedings of this meeting and direct that a copy be forwarded to Mrs. Hall with an expression of our great sympathy.

October 14, 1942.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

RECENT ADDITIONS.

AERONAUTICS.

JONES, BRADLEY. Elements of Practical Aerodynamics. Third Edition. 1942.
MANLY, G. B. Aviation from the Ground Up. 1942.
SHIELDS, BERT A. Theory of Flight and Aircraft Engines. Second Edition. 1942.
SUDDETH, JAMES H. Aircraft Engine Maintenance. 1942.
ZIM, HERBERT S. Parachutes. 1942.

AUTOMOTIVE ENGINEERING.

HELDT, P. M. Torque Converters or Transmissions. 1942.

BIOGRAPHY.

BERRY, ROBERT ELTON. Yankee Stargazer. The Life of Nathaniel Bowditch. 1941.

BIOLOGY AND BIOCHEMISTRY.

TAYLOR, HUGH STOTT, ERNEST O. LAWRENCE, AND IRVING LANGMUIR. Molecular Films. The Cyclotron and the New Biology. 1942.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

BECK, ADOLF. The Technology of Magnesium and its Alloys. 1941.

Chemical Rubber Publishing Company. Handbook of Chemistry and Physics. Twenty-Sixth Edition. 1942.

Chemical Society. Annual Reports on the Progress of Chemistry for 1941. Volume 38. 1942.

EGLOFF, GUSTAV, GEORGE HULLA, AND V. I. KOMAREWSKY. Isomerization of Pure Hydrocarbons. 1942.

FUSON, REYNOLD C., AND H. R. SNYDER. Organic Chemistry. 1942.

GRUSE, WILLIAM A., AND DONALD R. STEVENS. The Chemical Technology of Petroleum. Second Edition. 1942.

MEYER, KURT H. Natural and Synthetic High Polymers. 1942.

NORTON, F. H. Refractories. Second Edition. 1942.

RANDALL, MERLE, AND LEONA ESTHER YOUNG. Elementary Physical Chemistry. 1942.

Vanderbilt Rubber Handbook. Eighth Edition. 1942.

DICTIONARIES.

Webster's Dictionary of Synonyms. First Edition. 1942.

ELECTRICITY AND ELECTRIC ENGINEERING.

American Institute of Electrical Engineers. American Standard Definitions of Electrical Terms. 1942.

DEFOREST, LEE. Television Today and Tomorrow. 1942.

HENNEY, KEITH. Principles of Radio. Fourth Edition. 1942.

JORDAN, EDWARD C., AND OTHERS. Fundamentals of Radio. 1942.

MANLY, HAROLD P., AND L. O. GORDER. Drake's Cyclopedia of Radio and Electronics. Tenth Edition. 1942.

MILLMAN, JACOB, AND SAMUEL SEELY. Electronics. First Edition. 1941.

ENGINEERING.

CLARK, WILLIAM A., AND BRAINERD PLEHN. Materials Testing and Heat Treating. 1942.

ELBERFELD, JOHN. Strength and Properties of Materials. 1942.

GILLETTE, HALBERT P., AND JOHN C. BLACK. Road and Street Construction. 1940.

SUTHERLAND, HALE, AND HARRY LAKE BOWMAN. Structural Theory. Third Edition. 1942.

GEOLOGY.

PRATT, WALLACE E. Oil in the Earth. 1942.

GRAPHIC ARTS.

STERNBERG, HARRY. Silk Screen Color Printing. 1942.

HISTORY.

BRIDENBAUGH, CARL, AND JESSICA BRIDENBAUGH. Rebels and Gentlemen. 1942.

DAMPIER, SIR WILLIAM CECIL. A History of Science and its Relations with Philosophy and Religion. Third Edition, Revised and Enlarged. 1942.

MANUFACTURES.

American Welding Society. Welding Handbook. 1942.
HOYE, JOHN. Staple Cotton Fabrics. First Edition. 1942.
Lincoln Electric Company. Procedure Handbook of Arc Welding Design and Practice. Seventh Edition. 1942.

MATHEMATICS.

AITKEN, A. C. Statistical Mathematics. Second Edition. 1942.
KELLER, ERNEST G. Mathematics of Modern Engineering. Volume 2. 1942.

METEOROLOGY.

MIDDLETON, W. E. KNOWLES. Visibility in Meteorology. 1941.
SVERDRUP, H. U. Oceanography for Meteorologists. 1942.

MILITARY SCIENCE.

FAIRHURST, ISABELLE. Manual of High Explosives, Incendiaries and Poison Gases. 1942.
HOLMES, HARRY N. Strategic Materials and National Strength. 1942.
WAITT, ALDEN H. Gas Warfare. 1942.

MINING AND METALLURGY.

American Institute of Mining and Metallurgical Engineers. Transactions. Institute of Metals Division. 1942.
American Society for Metals. Controlled Atmospheres. 1942.
AUSTIN, I. B. The Flow of Heat in Metals. 1942.

NAVAL ARCHITECTURE AND NAVIGATION.

American Bureau of Shipping. Rules for the Classification and Construction of Steel Vessels. 1862–1942.
CORNELL, FELIX M., AND ALLAN C. HOFFMAN. American Merchant Seaman's Manual. Revised Edition. 1942.
KIELHORN, L. V. A Treatise on Compass Compensation. 1942.

PHILOSOPHY.

LOWINGER, ARMAND. The Methodology of Pierre Duhem. 1941.

PHYSICS.

BRIDGMAN, P. W. The Nature of Thermodynamics. 1941.
CULVER, CHARLES A. Musical Acoustics. 1941.
DAKE, H. C., AND JACK DEMENT. Ultra-Violet Light and its Applications. 1942.
HOWE, HARLEY. Introduction to Physics. First Edition. 1942.
LYNDE, CARLETON JOHN. Science Experiences with Home Equipment. 1937.
MERKER, JOHN C. High-Pressure Oil Hydraulics. First Edition. 1940.

SCIENTIFIC ESSAYS.

BLIVEN, BRUCE. The Men Who Make the Future. 1942.
EHRET, WILLIAM F., Editor. Physical Science. 1942.

SUGAR.

Sugar Reference Book and Directory. Volume XI. 1942.
ZERBAN, F. W., Editor. System of Cane Sugar Factory Control. 1942.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Distribution in the Rat of Injected Radioactivated Bacteria.—J. O. Ely. Some preliminary experiments on the distribution in rats of injected "radioactive bacteria" were reported earlier from this laboratory (*Notes from the Biochemical Research Foundation, Journal of The Franklin Institute*, **232**: 385, October, 1941). Since that time additional experiments with more species of bacteria have been performed. The results are presented here.

EXPERIMENTAL PROCEDURE.

The experimental procedure has been altered considerably since the previous report; therefore a complete description will be given here.

Bacteria were grown for 18 hours on agar slants of heart infusion broth to which had been added a small amount of radioactive sodium phosphate (phosphorus was the radioactive element) prepared from material irradiated in our cyclotron. The bacteria were removed from the surface of the medium with distilled water and then were washed 4 times, each time with approximately 400 volumes of distilled water. It was found that additional washing did not remove more radioactive material from the bacteria. For this reason it was assumed that the radioactive P remaining in the organisms had been incorporated into the compounds of the bacterial bodies. Williams, Sandholzer, and van Vorhees (*Journal of Bacteriology*, **39**: 19, 1940) have reported the incorporation of radioactive P in the nucleoproteins and phospholipids of *Escherichia coli*. After washing, the bacteria were suspended in 0.85 per cent. NaCl solution. The suspension was then filtered through a pad of wet cotton in order to remove clumps and obtain a uniform suspension.

Albino rats were used. Injection of the bacteria was made by the following routes: (*a*) intravenously into a tail vein, (*b*) intraperitoneally in the mid-abdominal area, (*c*) intrathoracically near the sternebra, care being exercised not to injure the organs within, (*d*) intramuscularly into the muscles

of the hind leg, (*e*) subcutaneously into the right groin area. At selected periods of time after injection the animals were killed by decapitation and the tissues removed for use.

Samples of the tissues were ashed by heating in micro-kjeldahl flasks with a mixture of sulfuric and nitric acids, with the addition where necessary of superoxol. From the digested samples the P was isolated as magnesium pyrophosphate (*Scott's Standard Methods of Chemical Analysis*, 1925, 1: 695, Van Nostrand Co., Inc.). The radioactivity of the magnesium salt was measured by means of a Geiger-Mueller counter.

In this work it is assumed that the relative distribution of the bacteria in the animal is the same as the distribution of the radioactive P, the P being a component of the bacteria. Under this premise it is necessary to assume that during the relatively short experimental periods used there is no appreciable lysis of the bacteria with a liberation of the radioactive phosphorus compounds. Giving support to this assumption is the different distribution of P_{32} when injected as radioactive sodium phosphate from the distribution when it is injected as "radioactive bacteria," shown in Table I.

THE RELATIVE DISTRIBUTION OF BACTERIA IN THE RAT AFTER INTRAVENOUS INJECTION.

Species of Bacteria Used:

1. *Bacillus megatherium*
2. *Bacillus palustris*
3. *Bacillus subtilis*
4. *Eberthella enterica*
5. *Eberthella typhi*
6. *Escherichia coli*
7. *Gaffkya tetragena*
8. *Mycobacterium phlei*
9. Pneumococci, type II, rough strain
10. *Proteus vulgaris*
11. *Pseudomonas aeruginosa*
12. *Pseudomonas fluorescens*
13. *Serratia marcescens*
14. *Staphylococcus aureus*
15. *Streptococcus fæcalis*
16. *Klebsiella rhinoscleromatis*

RESULTS.

In nearly every experiment the liver, as an organ, removed from the blood more of the bacteria than did either the spleen or lungs. Exception to this was *Bacillus megatherium* of which the lungs accumulated more in 5, 10, 15, and 20 minutes than did the liver, and in 30 minutes nearly as many.

The relative distribution after intravenous injection in the liver, spleen, kidney, brain, testes, and lungs was determined with *Es. coli*, *Eb. typhi*, *Staphylococcus aureus*, and *Bacillus subtilis*. It was found that during the experimental periods used, the liver, spleen, and lungs accumulated nearly all of the injected bacteria with only comparatively small proportions being found in the other tissues studied. This is shown in Table I. For this reason the determinations were thereafter confined largely to the liver, spleen, and lungs.

TABLE I.

The Relative Distribution of Radioactive Bacteria (and of Radioactive Sodium Phosphate) in the Rat after Intravenous Injection.

Rat No.	Wt.	Species of Bacteria.	Vol. of Inocula. Cc.	Expt'l Period, Min.	Liver.	Spleen.	Kidney. Both.	Brain.	Testes.	Tibia.	Lungs.
1	151	*Staph. aureus*	1.0	30	908	49	4	5	0	3	204
2	167	*Staph. aureus*	2.0	30	821	96	6	3	2	0	129
3	148	*Es. coli*	2.0	30	1,087	417	14	5	7	10	65
4	140	*Es. coli*	1.0	30	334	106	27	1	5	4	16
5	144	*Es. coli*	1.0	30	2,497	483	13	2	5	9	136
6	171	*Eb. typhi*	1.0	30	1,198	285	—	0	0	3	8
7		Sodium phosphate	0.8	30	9,845	1,017	4,700	71	415	7,335	1,020
8		Sodium phosphate	0.8	30	11,849	752	5,580	105	296	4,517	1,667
9	153	*B. subtilis*	1.0	30	1,097	41	3	0	2	3	146
10	166	*Eb. typhi*	1.0	10	655	203	7	1	0	1	4
11	132	*Eb. typhi*	1.0	30	837	292	5	1	0	2	7
12	140	*B. subtilis*	1.0	30	502	20	17	10	2	5	327
13	196	*Eb. typhi*	1.0	10	1,425	292	21	4	1	12	131
14	164	*Eb. typhi*	1.0	30	3,483	548	25	0	1	21	83
15	147	*B. subtilis*	1.0	5	700	18	14	7	3	3	120
16	154	*B. subtilis*	1.0	10	417	36	14	2	2	4	110

After intravenous injection into rats it was found that the species of bacteria studied may be divided into 3 groups in regard to their distribution under the conditions of these experiments: (1) The species of which the organisms are large

or form clumps or chains in suspensions; these accumulated to a greater degree in the lungs than in the spleen. (2) The species of which the organisms are small and form chains or clumps to a mild degree or not at all; these accumulated in the spleen to a greater extent than in the lungs. (3) Those species of which the organisms are small and are irregular in the degree of chain and clump formation; these showed little difference in relative amounts which accumulated in the lungs and spleen.

The relative average size of the bacteria of the different species of the 3 groups is shown in Fig. I. The average size was determined by measuring many enlarged images of the organisms photographed in stained smears prepared from suspensions similar to those used for injections into the experimental animals. The observations on the clumping and chain units were made on smears of bacteria prepared as for injection into the animals, i.e. suspended in 0.85 per cent. sodium chloride solution. Smears were then made and stained. In order to approximate more nearly the conditions that might be present in the blood, bacteria prepared for injection were added to rat blood serum in the proportion of 1 part of bacterial suspension to 6 parts of serum. The suspensions were shaken, and then examined under dark field illumination. It was concluded from this examination that the stained smears represented very well the conditions present in the suspensions injected, and probably also the conditions after introduction into the blood stream.

The average size of the bacteria when considered with microscopic examinations suggested that the distribution of these organisms, in regard to the relative amounts found in the lungs and spleen, is determined by size, chain formation and clumping. In other words, single bacteria, chains, or clumps are removed as units. After injection into a vein of the tail of the rat bacteria pass through the liver, lungs, and spleen in the order named. If the units are sufficiently large they will be caught and held in the liver. Of those units passing through the liver the larger ones will be filtered out by the lungs provided the meshes of the lungs are smaller than those of the liver. Those units escaping the lungs will be caught in the spleen provided it is a sufficiently fine filter.

FIGURE I.

THE RELATIVE SIZE OF BACTERIA

Group I

Those of which, after intravenous injection, greater numbers are found in the lungs than in the spleen.

(1) Bacillus megatherium
(2) Bacillus palustris
(3) Bacillus subtilis
(4) Eberthella enterica
(5) Staphylococcus aureus
(6) Mycobacterium phlei

Group II

Those of which, after intravenous injection, greater numbers are found in the spleen than in the lungs.

(1) Proteus vulgaris
(2) Escherichia coli
(3) Eberthella typhi
(4) Pseudomonas aeruginosa

Group III

Those of which, after intravenous injection, approximately equal numbers are found in the lungs and in the spleen.

(1) Gaffkya tetragena
(2) Pneumococcus, Type II, rough
(3) Pseudomonas fluorescens
(4) Streptococcus faecalis
(5) Klebsiella rhinoscleromatis
(6) Serratia marcessans

The average relative amounts of bacteria found per gram (wet weight) in the liver, lungs, and spleen after intravenous injection are shown in Table II.

The data for groups I and II indicate the spleen to be a finer filter than the lungs. It may be seen that in the case

TABLE II.

The Average Amounts of Bacteria Found per Gram (Wet Weight) of Livers, Lungs, and Spleens of Rats after Intravenous Injection.

Group No.	Number of Rats.	Liver.	Lungs.	Spleen.
I	49	35	87	10
II	27	7	2	10
III	25	30	11	10

of group I, in which are found the largest units, the lungs accumulated a far greater proportion of the bacteria than was found in the spleen, whereas of group II, in which are found the smallest units, the spleen accumulated about 5 times as many as did the lungs. Of group III, with units of intermediate size, there was little difference in the numbers accumulated by the lungs and spleen.

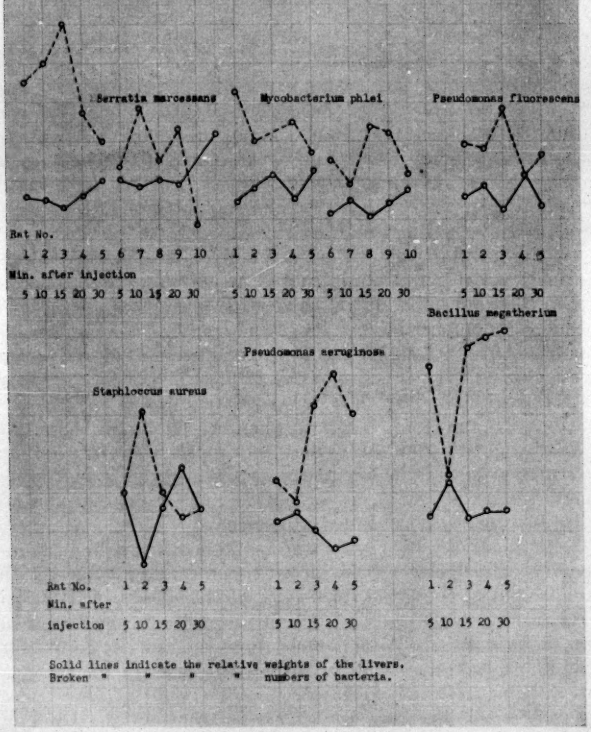
If removal of bacteria from the blood stream were purely a mechanical filtration it should be possible to determine from the data in Table II the relative fineness of the liver as a filter as compared to the lungs and spleen. This is possible so far as group I and group II are concerned, but not in the case of group III, because in this group the percentage of the total accumulated by the liver is greater than in the cases of groups II and I. On the mechanical filtration basis this would indicate that the units of group III were larger than those of group I, which is not true. This suggests that other factors may be active as far as the liver is concerned, and possibly also with the lungs and spleen.

As illustrated by Fig. II the numbers of bacteria accumulated by the liver per gram of wet tissue varied inversely with the weight of the livers for several of the species of bacteria. This indicates the passages are larger in the heavier than in the lighter livers. A similar relationship was not found for the lungs, but was found to be partially true for the spleens, there being frequent exceptions.

So far as concerns the relative amounts of the bacteria which accumulated in the livers, lungs, and spleens, as organs, the several species of bacteria fall into the same 3 groups as when the results were calculated on the basis of the unit of wet weight. Representative of the three groups based upon

Figure II

Comparison of the relative weights of livers of rats and the relative amounts of bacteria per gram (wet weight) after intravenous injection.


Serratia marcesans
Mycobacterium phlei
Pseudomonas fluorescens
Rat No.
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5
Min. after injection
5 10 15 20 30 5 10 15 20 30
5 10 15 20 30 5 10 15 20 30
5 10 15 20 30
Bacillus megatherium
Pseudomonas aeruginosa
Staphlococcus aureus
Rat No. 1 2 3 4 5
1 2 3 4 5
1 2 3 4 5
Min. after injection
5 10 15 20 30
5 10 15 20 30
5 10 15 20 30

Solid lines indicate the relative weights of the livers.
Broken " " " " numbers of bacteria.

Figure III

The relative distribution of Staphlococcus aureus in the spleens, lungs and livers of rats after intravenous injection.

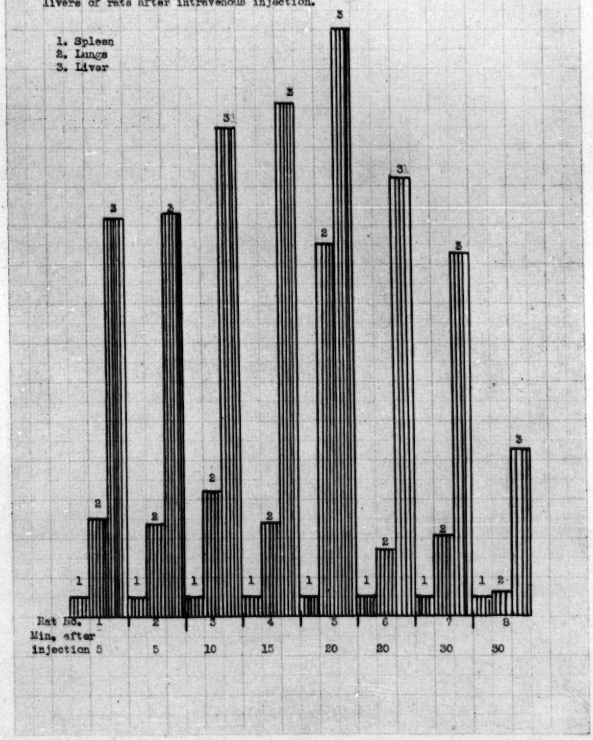


Figure IV

The relative distribution of Escherichia coli in the spleens, lungs and livers of rats after intravenous injection.

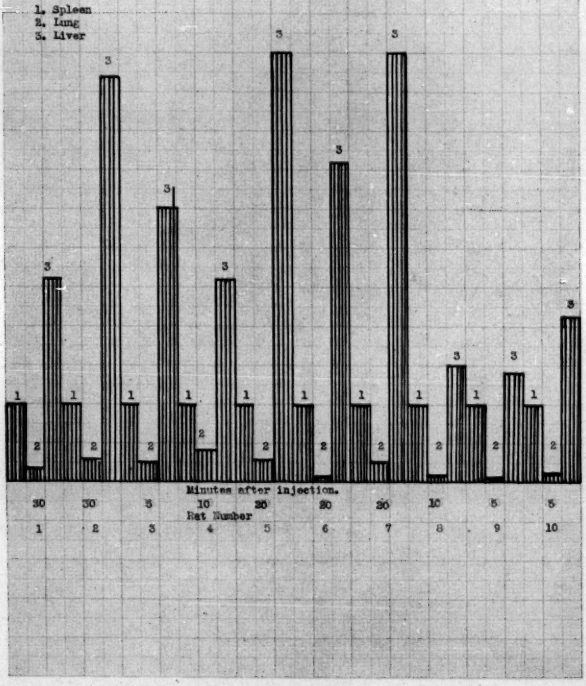
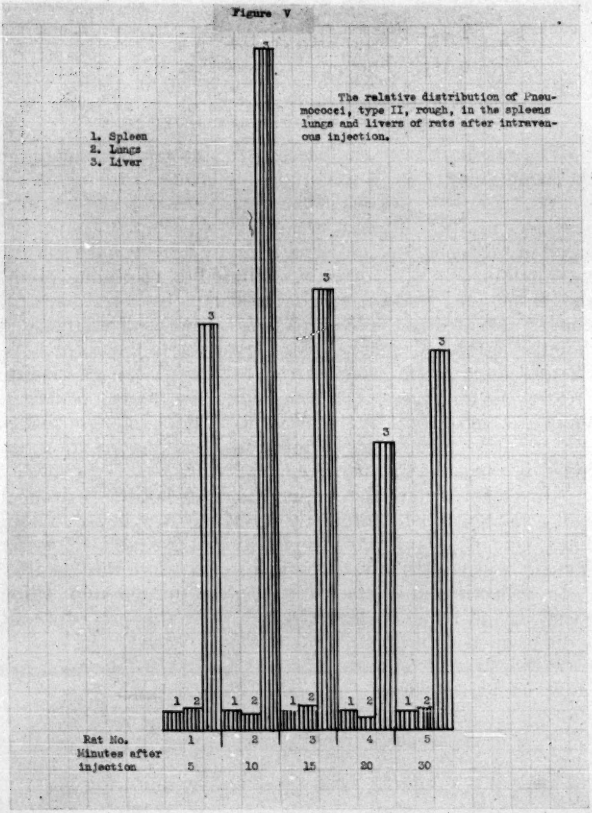


Figure V


1. Spleen
2. Lungs
3. Liver
The relative distribution of Pneumococci, type II, rough, in the spleens lungs and livers of rats after intravenous injection.
1 2 3
1 2 3
1 2 3
1 2 3
1 2 3
Rat No. 1 2 3 4 5
Minutes after injection 5 10 15 20 30

their accumulation in the liver, lungs, and spleen as whole organs are:

Group I. *Staphylococcus aureus* (see Fig. III)
Group II. *Escherichia coli* (see Fig. IV)
Group III. Pneumococci (see Fig. V)

THE RELATIVE DISTRIBUTION OF BACTERIA IN THE RAT AFTER INTRAPERITONEAL AND SUBCUTANEOUS INJECTION.

The preparation of the bacteria for injection was made in the same manner as in the studies following intravenous injection. Aliquot portions of the organs and tissues used were obtained in the following manner: (1) Liver. The entire organ was digested to a uniform liquid by heating with sulfuric and nitric acids. This digest was diluted to a suitable volume and an aliquot portion taken for complete digestion. (2) Lungs. Treated as the liver. (3) Intestines. This included the entire intestinal tract below the lower end of the æsophagus. The mesentery and fat were cut from the intestines with scissors. Treated as the liver. (4) Omentum, mesentery, pancreas, lymphatic tissue, etc., of the peritoneal cavity. These tissues were grouped together as a unit and an aliquot portion obtained in the same manner as for the liver.

After intraperitoneal injection the organs of the peritoneal cavity were washed thoroughly with 0.85 per cent. sodium chloride solution prior to use. The tissues used from the thoracic cavity after intrathoracic injection were washed also.

In collecting fluid the cavities were opened and saline placed in and removed several times. By this procedure all of the original fluid present was removed for use.

Species of Bacteria Used:

	Experimental Period, Hours.
1. *Bacillus subtilis*	1
2. *Staphylococcus aureus*	1
3. *Clostridium tetani*	1
4. *Escherichia coli*	1
5. *Eberthella typhi*	0.5 and 1

RESULTS.

The results varied with the organisms used and with the route of injection. It was found that after intraperitoneal in-

jection, the fluid in the abdominal cavity after a period of 1 hour contained approximately 40 to 60 per cent. of the total amount of bacteria found in all of the tissues examined and the unabsorbed fluid. The liver, mesentery, etc. and the

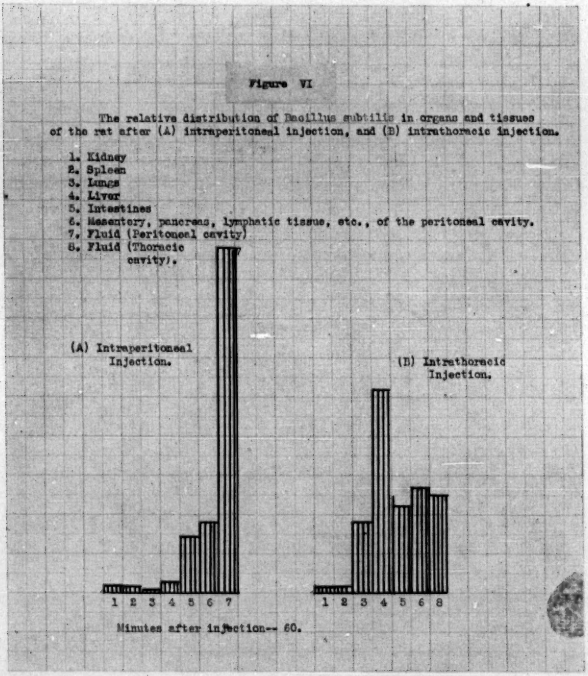


Figure VI
The relative distribution of Bacillus subtilis in organs and tissues of the rat after (A) intraperitoneal injection, and (B) intrathoracic injection.
1. Kidney
2. Spleen
3. Lungs
4. Liver
5. Intestines
6. Mesentery, pancreas, lymphatic tissue, etc., of the peritoneal cavity.
7. Fluid (Peritoneal cavity)
8. Fluid (Thoracic cavity).
(A) Intraperitoneal Injection.
(B) Intrathoracic Injection.
1 2 3 4 5 6 7
1 2 3 4 5 6 8
Minutes after injection-- 60.

intestines contained relatively large amounts. The spleen, lungs and kidneys contained relatively small amounts.

After intrathoracic injection, large numbers of the bacteria were found in the connective tissue support of the pericardium. The fluid in the thoracic cavity always contained relatively few bacteria compared to the numbers found in the abdominal

fluid after intraperitoneal injection. It was observed also that injected fluid was removed much more rapidly from the thoracic cavity than from the peritoneal cavity.

The lungs always contained somewhat more bacteria after intrathoracic injection than after intraperitoneal injection.

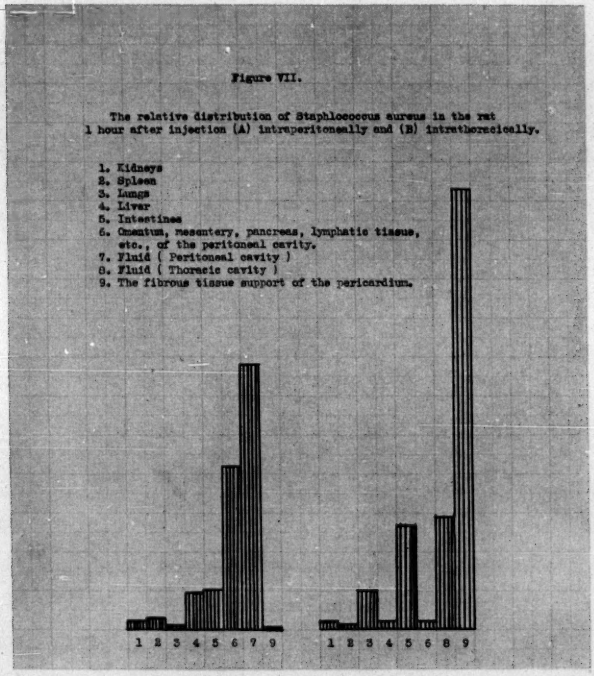


Figure VII.
The relative distribution of Staphlococcus aureus in the rat 1 hour after injection (A) intraperitoneally and (B) intrathoracically.
1. Kidneys
2. Spleen
3. Lungs
4. Liver
5. Intestines
6. Omentum, mesentery, pancreas, lymphatic tissue, etc., of the peritoneal cavity.
7. Fluid (Peritoneal cavity)
8. Fluid (Thoracic cavity)
9. The fibrous tissue support of the pericardium.
1 2 3 4 5 6 7 9
1 2 3 4 5 6 8 9

However, the amount was not sufficiently greater to denote the lungs as a main route of absorption of the bacteria from the thoracic cavity unless they passed on immediately to other points.

Representative examples are given in Figs. VI and VII.

THE RELATIVE DISTRIBUTION OF BACTERIA IN THE RAT AFTER SUBCUTANEOUS AND INTRAMUSCULAR INJECTION.

The bacterial suspensions were prepared as for the previous experiments, and the tissues were treated in the same manner as previously noted.

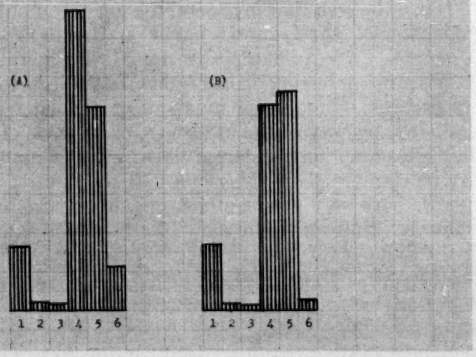
Species of Bacteria Used:

	Experimental Period, Hours.	Route of Injection.
1. *Eberthella typhi*	2	Intramuscularly and subcutaneously
2. *Staphylococcus aureus*	1.5	Intramuscularly and subcutaneously
3. *Bacillus subtilis*	1.75	Intramuscularly and subcutaneously
4. *Eberthella enterica*	2	Intramuscularly and subcutaneously
5. *Mycobacterium phlei*	1	Intramuscularly
6. *Mycobacterium phlei*	1.5	Subcutaneously
7. *Escherichia coli*	1	Intramuscularly
8. *Escherichia coli*	1.5	Subcutaneously

Figure VIII

The relative distribution of Bacillus Subtilis in the rat 1.75 hours after (A) intramuscular injection and 1.75 hours after (B) subcutaneous injection.

1. Kidneys
2. Spleen
3. Lungs
4. Liver
5. Intestines
6. Omentum, mesentery, pancreas, lymphatic tissue etc., of the peritoneal cavity.



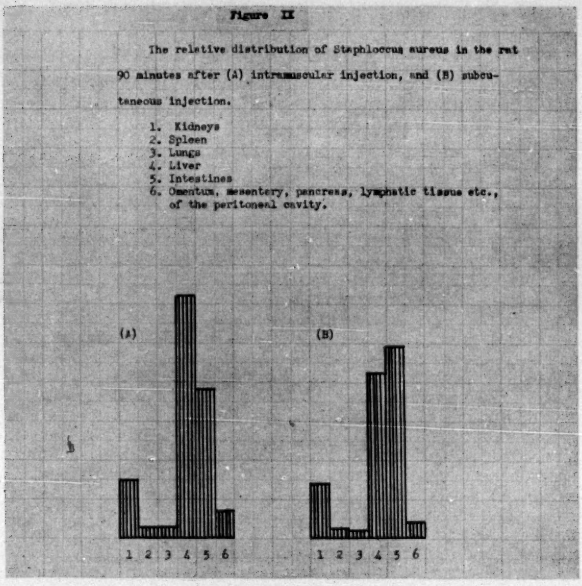


Figure II

The relative distribution of Staphloccus aureus in the rat 90 minutes after (A) intramuscular injection, and (B) subcutaneous injection.

1. Kidneys
2. Spleen
3. Lungs
4. Liver
5. Intestines
6. Omentum, mesentery, pancreas, lymphatic tissue etc., of the peritoneal cavity.

RESULTS.

It may be seen that in general the liver accumulated a large proportion of the bacteria found in the tissues examined. In general, large amounts were found also in the intestines. This is an interesting feature that is subject to at least 2 interpretations: (1) large numbers of the bacteria may be eliminated by way of the intestines, or (2) lysis of bacteria in the tissues may have occurred liberating P compounds which may be eliminated by this route. If the latter possibility be true the data may not be a true measure of the bacterial distribution. Subject to the same interpretations are the results for the kidneys for 3 of the bacterial species: *Bacillus subtilis*, *Staphylococcus aureus* and *Eberthella typhi*. These are questions that need further investigation before any conclusion can be reached.

Representative examples are given in Figs. VIII and IX.

BOOK REVIEWS.

INDUSTRIAL CHEMISTRY, by Emil Raymond Riegel, Ph.D. Fourth edition, 861 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1942. Price $5.50.

This much-used reference work is now in its fourth edition. It brings up-to-date the vast and diversified field known as industrial chemistry in the form, and to the extent, that is most useful to the student, general reader, and as a source of general information to the engineer and scientist. It comes to hand at a time when the chemist and chemical engineer are among those to whom the nation turns for the development and manufacture of products to win the war.

The plan of presentation is little changed from previous editions. It begins with a description of the processes of sulfuric acid manufacture and proceeds through the subjects of nitric acid, hydrochloric acid and the manufacture of soda, nitrogen products, phosphates, fertilizers, cement and plaster, glass, fuels and power production, refrigeration, water and sewage, gas manufacture, distillation of wood, products of the electrothermal furnace, electrolysis, alcohol, textiles, sugar, petroleum, dyes, fats and waxes, paints and varnishes, soap, explosives, plastics, leather, glue, photography, rubber, patents, engineering machinery, materials of construction, instruments of control, iron and steel, and other metals. There are many changes and revisions, large and small. The work is profusely illustrated with photographs, curves and drawings, the latter including flow sheets of much value.

The style in which the material is presented is impressive. Generally, it involves, (1) a broad statement about the industry, its size and importance, (2) a description of the process or principle processes with drawings of the apparatus, (3) by-products, (4) the processing of by-products. At the ends of chapters can be found a list of patents, a series of problems, and a list of references for further reading.

The author and his collaborators have produced, in this book, a valuable tool which has wide potential use. Its pages are full of information, quickly found and in an easily understandable form. It is a notable contribution to the literature.

R. H. OPPERMANN.

MICROWAVE TRANSMISSION, by J. C. Slater. First edition, 309 pages, illustrations, 16 × 24 cms. New York, McGraw-Hill Book Company, 1942. Price $3.50.

As one progresses in the study of electromagnetic waves from audio frequencies to radio frequencies to short waves, there is unfolded a continual array of new and curious effects even though but few new principles are needed for understanding. Microwave transmission offers even greater fascination for this is so different, and the effects are so startling. Conductors in the microwave region are very different: coaxial lines, consisting of a rod carried concentrically within a hollow pipe, or even simply a hollow pipe alone. In the case of the line,

current flows down the rod and back along the pipe. The pipe is different in that current flows in one direction in the inside wall and in the other direction through the outside wall. Though the two walls are connected electrically to each other, this condition does not lead to a short circuit. Microwaves are so different from longer waves that a new technique is needed.

This book, the first of its kind, is devoted to transmission almost exclusively, and merely refers briefly to power sources and receivers. It begins with the theory of transmission, which is developed without reference to any particular type of line and using ideas of electric circuits alone. From this theory, the student is lead to the use of Maxwell's equations and the natural extension of the study of plane waves. Then, the thread moves on to rectangular wave guides, and the more complicated types of transmission lines, the most important of which is the coaxial cable. The general problem is first set up before consideration is given in detail to the parallel wire transmission line and the coaxial cable.

In contrast with what has been treated up to this point—disturbances propagated along one definite direction—the next chapter deals with an antenna in free space which sends out radiation in all directions. Here the spherical waves are first investigated, then one of the particular advantages of microwaves, the ability to direct the waves in a rather sharp beam. These chapters on the nature of antennas provide the necessary information for considering the case in which one part of a transmission line is a coaxial cable and the other part a hollow pipe, since the input from the coaxial into the pipe is of the general nature of an antenna and the situation is closely related to that of radiation from antennas.

The way in which the author approaches the subject and his "carry-through" are commendable and reflect careful decisions. The work combines that which has appeared in several periodicals at various times with other knowledge, so that in this compact form it is of interest to the student such as the physicist and engineer.

R. H. OPPERMANN.

ASTRONOMY, MAPS, AND WEATHER, by C. C. Wylie. 449 pages, illustrations, 16 × 24 cms. New York and London, Harper & Brothers Publishers, 1942. Price $3.00.

This book has been prepared at the request of the Army Air Corps Flying Training Command to meet the needs of a supplemental course for college students for the Army Air Corps Reserve. It reflects careful selection of subjects, a thorough knowledge of the essentials necessary, and experience in fusing the subjects together to form a logical progression. As the title indicates, a variety of subjects are covered, intended to give sufficient understanding to airmen.

At the outset, there are covered the celestial sphere, the constellations, telescopes, the earth, motions of the earth, and the seasons and the calendar—a general discourse basic for astronomy and navigation. Next the weather, clouds, and weather forecasting are taken up—definitely of practical value. The broad subject of navigation is next taken up with the discussion divided into parts on maps, time, and celestial navigation. And the last part of the book is devoted to celestial bodies—the moon, the planets, the sun, the stars, etc.

The book is profusely illustrated with photographs and drawings. Every effort is made to make the material readable and understandable. This includes

the exercises at the ends of chapters. There is a subject index in the back which adds to the value of the book for reference.

R. H. OPPERMANN.

ELECTROPHORESIS OF PROTEINS AND THE CHEMISTRY OF CELL SURFACES, by Harold A. Abramson, Laurence S. Moyer, and Manuel H. Gorin. 341 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1942. Price $6.00.

Of the four general methods of making measurements leading to the calculation of electrokinetic potentials, the electrophoretic technics seem to be the most generally useful for the investigation of protein and biological surfaces. Once this was recognized, rapid advances resulted and our understanding of systems containing proteins and protein-like compounds has been greatly increased. Nor, is the end yet in sight. The subject is one of diverse interests covering the fields of chemists, physicists, biologists, and physicians. It comprises a borderland of the sciences. A means of coördination of these interests, of summarizing the present state of the subject, of acquiring a picture of all phases of the subject—all are the purposes of this book.

Proceeding from the simple to the more complex, the book may be divided into two parts. The first comprises the first four chapters. It lays the foundation upon which the rest is built. Chapter I contains an elementary presentation in non-mathematical form of the general principles of electrophoretic migration in liquids, which is followed by a brief historical background of experiments. Then in Chapter III, there are discussed certain methods of most value for the quantitative investigation of dissolved proteins, protein surfaces and cell surfaces; and Chapter IV reviews a restricted number of experiments dealing with the electric mobilities of dissolved proteins and particles coated with protein. This especially prepares the reader to follow the discussion taken up in the beginning of the second part of the book in connection with the theories of electric migration. Chapter VI illustrates the way in which proteins can be used to test the general theories of electrokinetics, conductance, and protein behavior. The book is so planned that the reading of the second part can be deferred without impairing the understanding of the material discussed elsewhere.

The rest of the book is primarily descriptive—dealing with serum and plasma, reactions of antibodies and antigens, interactions of proteins and mixtures, interactions of proteins at surfaces, enzymes and hormones, latex, surface chemistry of cells, and miscellaneous electrophoretic investigations of biological interest. There is a name and subject index in the back.

The book fills a need of many who are interested in the subject. It most certainly is an aid in the investigation of the surface chemistry of living cells.

R. H. OPPERMANN.

ENGINEERING MECHANICS, by Frank L. Brown. Second edition, 503 pages, illustrations, 16 × 24 cms. New York, John Wiley & Sons, Inc., 1942. Price $4.00.

No subject is more basic to engineering than mechanics. It enters into every branch of engineering, and provides a means of understanding theoretical as well as practical situations. Many times, mechanics are used as analogies

to somewhat foreign processes, which is a rather useful side-line application of mechanical knowledge. The importance of the subject is such that extreme care must be exercised in the compilation of a text for use of undergraduate students. Nothing must be sacrificed in the way of essentials to a complete and thorough understanding, yet the study must be limited in rigorousness and length.

Engineering Mechanics is in its second edition, the first having been published in 1931. This indicates an advantage in experience in textual presentation of the subject which is reflected in the second edition. The book is first divided into a part on statics and a part on kinematics and kinetics. Under the former, and after a coverage of general principles, coplanar force systems are discussed—their resultants and equilibrium, as well as the resultants and equilibrium of non-coplanar force systems. This is followed by treatments under the headings of plane trusses, friction suspended cables and center of gravity. Under the study of motion and kinematics, the fundamental laws of motion are stated and discussed with reference to the single particle. The transition from the laws for the particle to the practical formulas for the finite body is then made by means of summations and other algebraic processes.

The sequence of topics is logical and shows a careful layout of the method before actual presentation. There is a smooth progression from one topic to another. The author has used a very free style resulting generally in first descriptive matter, then illustrative matter usually in the solution of a problem or problems, followed by a discussion and problems for exercise, which are rather numerous and about ½ of which are provided with answers. The treatment is not sugar-coated, but great effort is made in strategic presentation and simplification. It represents a rigorous analytical course for the student of engineering.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Electrophoresis of Proteins and the Chemistry of Cell Surfaces, by Harold A. Abramson, Laurence S. Moyer, and Manuel H. Gorin. 341 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1942. Price $6.00.

Astronomy, Maps, and Weather, by C. C. Wylie. 449 pages, illustrations, 16 × 24 cms. New York & London, Harper & Brothers Publishers, 1942.

Lectures on Machine Design, by Lewis F. Moody. 75 pages, illustrations, 21 × 27 cms. Ann Arbor, Edwards Brothers, Inc., 1942. Price $1.75.

Poisson's Exponential Binomial Limit, by E. C. Molina. Table I—Individual Terms, Table II—Cumulated Terms. 47 pages, 20 × 28 cms. New York, Van Nostrand Company, Inc., 1941. Price $2.75.

Natural Trigonometric Functions, by Howard Chapin Ives, C. E. Second edition, 351 pages, 18 × 26 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Limited, 1942. Price $9.00.

Photography, Its Principles and Practice, by C. B. Neblette. Fourth edition, 865 pages, illustrations, 16 × 24 cms. New York City, D. Van Nostrand Company, Inc., 1942. Price $7.50.

Natural History with a Camera, by L. W. Brownell. 292 pages, illustrations, 16 × 24 cms. Boston, American Photographic Publishing Company, 1942. Price $3.75.